Convergence analysis of discretization schemes for the Navier Stokes equations on general grids

Raphaèle Herbin

Université d'Aix Marseille

Colloque en l'honneur de Matania Ben-Artzi
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Why Navier Stokes ?

Publications results for

"Author=(BenArtzi, M*) AND Title=(Navier-Stokes)"

Ben-Artzi, Matania; Croisille, Jean-Pierre; Fishelov, Dalia Convergence of a compact scheme for the pure streamfunction formulation of the unsteady system. SIAM J. Numer. Anal. 44 (2006), no. 5, 1997–2024 (electronic). (Reviewer: Filippo Maria Denaro) 76M20 (65M06 76D05)

Ben-Artzi, Matania; Croisille, Jean-Pierre; Fishelov, Dalia; Trachtenberg, Shlomo A pure-compact scheme for the streamfunction formulation of Navier-Stokes equations. J. Comput. Phys. 205 (2005), no. 2, 640–664. (Reviewer: Nikolay P. Moshkin) 76D05 (65M06 76M20)

Fishelov, D.; Ben-Artzi, M.; Croisille, J.-P. A compact scheme for the streamfunction formulation of Navier-Stokes equations. Computational science and its applications—ICCSA 2003. Part I, 809–817, Lecture Notes in Comput. Sci., 2667, Springer, Berlin, 2003. 65M06 (76D05 76M20)

Ben-Artzi, Matania Planar Navier-Stokes equations: vorticity approach. Handbook of mathematical fluid dynamics, Vol. II, 143–167, North-Holland, Amsterdam, 2003. (Reviewer: Sergey A. Sazhenkov) 35Q30 (76D03 76D05)

Ben-Artzi, Matania; Fishelov, Dalia; Trachtenberg, Shlomo Vorticity dynamics and numerical resolution of Navier-Stokes equations. M2AN Math. Model. Numer. Anal. 35 (2001), no. 2, 313–330. 76M23 (35Q30 65M99 76D17)

Ben-Artzi, Matania Global solutions of two-dimensional Navier-Stokes and Euler equations. Arch. Rational Mech. Anal. 128 (1994), no. 4, 329–358. (Reviewer: Yoshikazu Giga) 35Q30 (35B60 76C99 76D05)

Finite volume schemes in industrial codes

Staggered schemes

MAC scheme (e.g. pressure at center, velocities at edges) Harlow and Welsh Patankar, limited to simple geometries (structured grids) FLUENT, first version.

Ongoing work on unstructured grids.

Mathematical analysis requires small data condition (Nicolaides)

Collocated schemes

all unknowns located in the cell.

Complex 3D geometries, general meshes (SATURNE, FLUENT)

Lack of inf-sup condition, pressure stabilization needed.

Aim :
General non conforming meshes
Stability, accuracy, symmetry for the pure diffusion operator
mathematical proof of convergence of the scheme

Incompressible Navier-Stokes equations in primitive variables

$$-\nu\Delta\bar{u} + \nabla\bar{p} + \mathrm{div}(\bar{u}\otimes\bar{u}) = f \text{ in } \Omega$$

$$\mathrm{div}\bar{u} = 0 \text{ in } \Omega, \qquad \bar{u} = 0 \text{ on } \partial\Omega$$

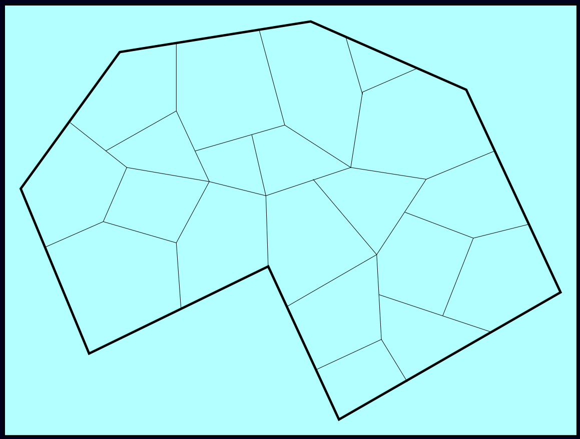
$$\int_\Omega \bar{p}(x)\mathrm{d}x = 0 \text{ (to enforce uniqueness of the pressure)}$$

Discretization on general grids

Convection terms :
centred or upwind FV scheme

Pressure gradient :
dual of the discrete FV divergence operator

viscous terms :
gradient scheme: "SUCCES" (Scheme Using Conservativity and Consistency Estimates for Stabilization)



Approximation of the diffusion terms - the orthogonal case

1/ Discretization of the balance form

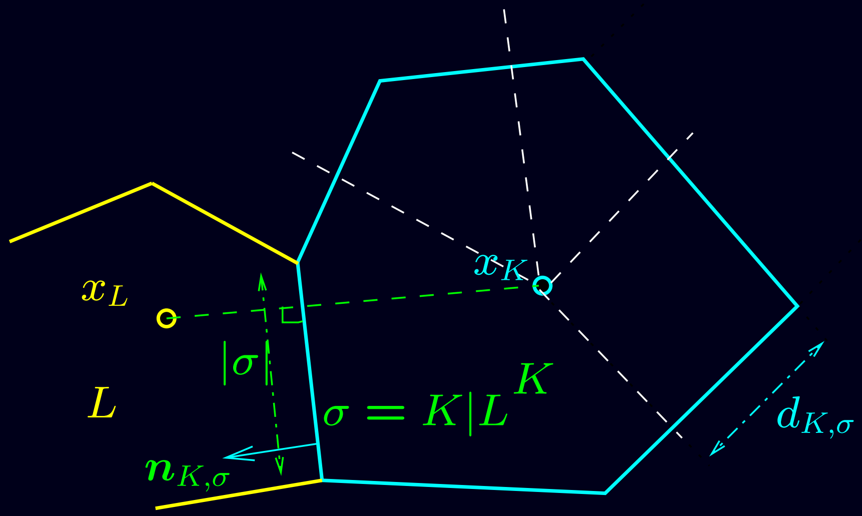
$$\int_K -\Delta u = -\int_{\partial K} \nabla u \cdot \mathrm{n}_K = \sum_{\sigma \in K} -\int_\sigma \nabla u \cdot n_{K,\sigma}$$

2/ Approximation of the normal fluxes

$$-\int_\sigma \nabla u \cdot n_{K,\sigma} \approx |\sigma| \frac{u_K - u_L}{d(x_K, x_L)}$$

$$\Downarrow$$

consistency of the numerical fluxes, needed for the scheme to converge.



FV schemes for general grids or anisotropic problems

Several attempts to build consistent fluxes on non-orthogonal grids

(or to discretize $-\nabla \cdot (A \nabla u) = f$):

- Multi point schemes (Aavatsmark et al., Le Potier)
- Discrete duality finite volume schemes (Hermeline, Domelevo, Omnes)
- Mimetic finite difference (Brezzi, Lipnikov, Shashkov et al.)
- Stabilized gradient schemes (Eymard Gallouet H.)

$\vdots$

see FVCA5 benchmark, organized by RH and Florence Hubert:

http://www.latp.univ-mrs.fr/fvca5/

Construction of a consistent gradient

Mesh: $\Omega = \cup K\{K \in \mathcal{T}\}$, with interfaces $\{\sigma \in \mathcal{E}\}$.

For known values $(u_K)_{K\in\mathcal{T}}$ $(u_\sigma)_{\sigma\in\mathcal{E}}$, construct a discrete gradient $\nabla_{\mathcal{T}} u$, constant on each cell K.

If u is linear on K, then: $u(\mathrm{x}_\sigma) - u(\mathrm{x_K}) = (\nabla \mathrm{u})_{\mathrm{K}} \cdot (\mathrm{x}_\sigma - \mathrm{x_K})$

On a general mesh: $u_\sigma - u_K \approx \nabla u \cdot (\mathrm{x}_\sigma - \mathrm{x}_K)$

Reconstruct $(\nabla_{\mathcal{T}} u)_K$ from these values ?

$$\int_{\partial K} \mathbf{n_K} \mathbf{x}^t d\gamma(\mathbf{x}) \left(= \int_{\partial K} \mathbf{n_K} (\mathbf{x} - \mathbf{x}_K)^t d\gamma(\mathbf{x}) \right) = |K| Id \qquad \textbf{(MF)}$$

Proof : For $\mathbf{y}, \mathbf{z} \in \mathbb{R}^d$,

$$\int_{\partial K} \mathbf{y}^t \ \mathbf{n}_K \ \mathbf{x}^t \ \mathbf{z} = \int_{\partial K} [z \cdot \mathbf{x} \ \mathbf{y}] \cdot \mathbf{n}_K d\gamma(\mathbf{x}) = \int_{\partial K} \nabla \cdot [(\mathbf{x} \cdot \mathbf{z}) \ \mathbf{y}] = \mathbf{y} \cdot \mathbf{z} \ |K|.$$

If u is linear on K, (MF) $\Longrightarrow$

$$\begin{aligned} |K|(\nabla u)_K &= \int_{\partial K} (\mathbf{x} - \mathbf{x}_K) \cdot (\nabla u)_K \mathbf{n}_K \\ &= \sum_{\sigma \in K} |\sigma| (\mathbf{x}_\sigma - \mathbf{x}_K) \cdot (\nabla u)_K \mathbf{n}_{K,\sigma} \end{aligned}$$

Discrete gradient on K:

$$(\tilde{\nabla}_{\mathcal{T}} u)_K = \frac{1}{|K|} \sum_{\sigma \in K} |\sigma| (u_\sigma - u_K) \ \mathbf{n}_{K,\sigma}$$

A stability problem

Uniform rectangular mesh,

$$(\tilde{\nabla}_{\mathcal{T}} u)_{K_{i,j}} = \begin{pmatrix} \frac{1}{h_x}\left(u_{i+1/2,j} - u_{i-1/2,j}\right) \\ \frac{1}{h_y}\left(u_{i,j+1/2} - u_{i,j-1/2}\right) \end{pmatrix}$$

If $u_{i+1/2,j} = \frac{u_{i+1,j} + u_{i,j}}{2}$ **and** $u_{i,j+1/2} = \frac{u_{i,j+1} + u_{i,j}}{2}$

$$(\tilde{\nabla}_{\mathcal{T}} u)_{K_{i,j}} = \begin{pmatrix} \frac{1}{2h_x}\left(u_{i+1,j} - u_{i-1,j}\right) \\ \frac{1}{2h_y}\left(u_{i,j+1} - u_{i,j-1}\right) \end{pmatrix}$$

Consistent, but unstable... Idea: stabilize by a consistency error

Stable gradient

$$\tilde{\nabla}_K u = \frac{1}{|K|} \sum_{\sigma \in \mathcal{E}_K} |\sigma| (u_\sigma - u_K) \boldsymbol{n}_{K,\sigma}$$

$$R_{K,\sigma} u = \frac{\alpha_K}{d_{K,\sigma}} \Big(u_\sigma - u_K - \tilde{\nabla}_K u \cdot (\boldsymbol{x}_\sigma - \boldsymbol{x}_K) \Big)$$

$\nabla_{\mathcal{T}} u$ **constant in** $D_{K,\sigma}$ **and** $\nabla_{K,\sigma} u = \tilde{\nabla}_K u + R_{K,\sigma} u \, \boldsymbol{n}_{K,\sigma}$

Getting rid of interface values

$x_\sigma = \sum_K a_\sigma^K x_K$, $\Pi_\sigma u = \sum_K a_\sigma^K u_K$ **and replace** u_σ **by** $\Pi_\sigma u$

Approximate diffusive term

$$\bar{u}, \bar{v} \in H_0^1(\Omega),$$

$u, v \in H_{\mathcal{T}}(\Omega) =$ **piecewise constant functions on the grid cells,**

$$\int_\Omega \nabla\bar{u}\nabla\bar{v}\,\mathrm{d}\boldsymbol{x} \approx \int_\Omega \nabla_{\mathcal{T}}u\nabla_{\mathcal{T}}v\,\mathrm{d}\boldsymbol{x}$$

$\nabla_{\mathcal{T}}u$ **constant in** $D_{K,\sigma}$ **and** $\nabla_{K,\sigma}u = \tilde{\nabla}_K u + R_{K,\sigma}u\ \boldsymbol{n}_{K,\sigma}$

$$\tilde{\nabla}_K u = \frac{1}{|K|}\sum_{\sigma\in\mathcal{E}_K}|\sigma|(\Pi_\sigma u - u_K)\boldsymbol{n}_{K,\sigma}$$

$$R_{K,\sigma}u = \frac{\alpha_K}{d_{K,\sigma}}\Big(\Pi_\sigma u - u_K - \tilde{\nabla}_K u\cdot(\boldsymbol{x}_\sigma - \boldsymbol{x}_K)\Big)$$

Case of "super-admissible" meshes

If: $x_\sigma - x_K = d_{K,\sigma} n_{K,\sigma}$ **(triangles, rectangles, parallelepipeds)**

with appropriate choice of Π_σ and α_K

$$\Downarrow$$

$$\int_\Omega \nabla_{\mathcal{T}} u \cdot \nabla_{\mathcal{T}} v \mathrm{d}x = \sum_{\substack{\sigma \in \mathcal{E}_{\text{int}} \\ \mathcal{T}_\sigma = \{K,L\}}} \frac{|\sigma|}{d_{K,\sigma} + d_{L,\sigma}} (u_L - u_K)(v_L - v_K) + \sum_{\substack{\sigma \in \mathcal{E}_{\text{ext}} \\ \mathcal{T}_\sigma = \{K\}}} \frac{|\sigma|}{d_{K,\sigma}} u_K v_K$$

two points FV scheme: $-\int_K \Delta u \approx \sum_{\sigma \in \mathcal{E}_K} F_{K,\sigma} u$

with $F_{K,\sigma} u = \dfrac{|\sigma|}{d_{K,\sigma} + d_{L,\sigma}} (u_K - u_L)$

(Eymard Gallouët H. Handbook of Numerical Analysis, 2000)

Diffusive term convergence

Strong consistency of the gradient: for $\varphi \in C^2(\overline{\Omega}) \cap H_0^1(\Omega)$, $P_{\mathcal{T}}\varphi \in H_{\mathcal{T}}(\Omega)$ **s.t.** $(P_{\mathcal{T}}\varphi)_K = \varphi(x_K)$ **then** $\nabla_{\mathcal{T}} P_{\mathcal{T}}\varphi \to \nabla\varphi$ **in** $L^2(\Omega)^d$ **as** $h_{\mathcal{T}} \to 0$

Consequence : **If** $\|\nabla_{\mathcal{T}} u_{\mathcal{T}}\|_{L^2(\Omega)^d}$ **bounded**

$$\int_\Omega \nabla_{\mathcal{T}} u_{\mathcal{T}}(x) \cdot \nabla_{\mathcal{T}} P_{\mathcal{T}}\varphi(x)\mathrm{d}x \to \int_\Omega G \cdot \nabla\varphi(x)\mathrm{d}x \text{ as } h_{\mathcal{T}} \to 0$$

Question: $\exists$ **?** $\overline{u} \in H_0^1(\Omega)$ **;** $G = \nabla \overline{u}$ **and** $\bar{u} = \lim_{h_{\mathcal{T}} \to 0} u_{\mathcal{T}}$

Discrete norm:

$$\|u\|_{1,\mathcal{T}}^2 = \sum_{\sigma = K|L \in \mathcal{E}_{\text{int}}} \frac{|\sigma|}{d_{K,\sigma} + d_{L,\sigma}} (u_K - u_L)^2 + \sum_{\sigma \in \mathcal{E}_{\text{ext}}} \frac{|\sigma|}{d_{K,\sigma}} (u_K)^2.$$

$$\|u\|_{1,\mathcal{T}}^2 \leq \sum_{K \in \mathcal{T}} \sum_{\sigma \in \mathcal{E}_K} \frac{|\sigma|}{d_{K,\sigma}} (\Pi_\sigma u - u_K)^2 \leq C \|\nabla_{\mathcal{T}} u_{\mathcal{T}}\|_{L^2(\Omega)^d}^2$$

Discrete functional analysis

a/ Estimate on the translations in $L^1(\Omega)$: $\|u(\cdot+\xi)-u\|_{L^1(\mathbb{R}^d)} \leq \xi \|u\|_{BV}$

b/ Discrete Hölder $\|u(\cdot+\xi)-u\|_{L^1(\mathbb{R}^d)} \leq C\xi \|u_{\mathcal{T}}\|_{1,\mathcal{T}}$

c/ Kolmogorov th. $\rightsquigarrow$ convergence of $u_{\mathcal{T}}$ in L^1

d/ Discrete Sobolev $\rightsquigarrow$ estimate in $L^{\frac{2d}{d-2}}$ (3D) or $L^q,\ q<+\infty$ (2D)

c+d/ $\Longrightarrow$ convergence of $u_{\mathcal{T}}$ in L^2 to $\bar{u}$

e/ $\bar{u} \in H_0^1$

Hence: $u_{\mathcal{T}} \to \bar{u} \in H_0^1$ **in** $L^(\Omega)$

+ Weak convergence of $\nabla_{\mathcal{T}} u_{\mathcal{T}}$ to $\nabla \bar{u}$ in $L^2(\mathbb{R}^d)^d$

$$\Downarrow$$

If $\int_\Omega |\nabla_{\mathcal{T}} u|^2$ **bounded then** $\int_\Omega \nabla_{\mathcal{T}} u \nabla_{\mathcal{T}} P_{\mathcal{T}} \varphi \to \int_\Omega \nabla u \nabla \varphi.$

Back to NS: discrete convective terms

$\boldsymbol{u} \in \boldsymbol{H}_{\mathcal{T}}(\Omega)^d$ **discrete velocity,** $p \in H_{\mathcal{T}}(\Omega)$ **discrete pressure**

Mass conservation:

$$\int_K \mathrm{div}(\boldsymbol{u}) \approx \sum_{\sigma \in \mathcal{E}_K} \underbrace{\int_\sigma u \cdot \boldsymbol{n}_{K,\sigma}}$$

$$\approx \Phi_{KL}(\boldsymbol{u}, \lambda, p) = |\,\sigma\,|\, \Pi_\sigma \boldsymbol{u} \cdot \boldsymbol{n}_{K,\sigma} + \lambda_{K|L}(p_K - p_L)$$

General convection term:

$$\int_K \mathrm{div}(z\boldsymbol{u}) \rightsquigarrow \sum_{\sigma \in \mathcal{E}_K} \underbrace{\int_\sigma z\boldsymbol{u} \cdot \boldsymbol{n}_{K,\sigma}}$$

$$\approx \frac{z_K + z_L}{2} \Phi_{KL}(\boldsymbol{u}, \lambda, p)$$

In particular:

$$\int_K \mathrm{div}(u^{(i)}\boldsymbol{u}) \rightsquigarrow \sum_{\sigma\in\mathcal{E}_K} \underbrace{\int_\sigma u^{(i)}\boldsymbol{u}\cdot\boldsymbol{n}_{K,\sigma}} \tag{1}$$

$$\approx \frac{u_K^{(i)}+u_L^{(i)}}{2}\Phi_{KL}(\boldsymbol{u},\lambda,p) \tag{2}$$

Definition of discrete divergence

$$\mathrm{div}_K(z,\boldsymbol{u},\lambda,p) = \frac{1}{|K|}\sum_{\sigma\in\mathcal{E}_K,\mathcal{T}_\sigma=\{K,L\}} \Phi_{KL}(\boldsymbol{u},\lambda,p)z_\sigma$$

$$z_\sigma = \begin{cases} \textbf{centred choice} : \frac{z_K+z_L}{2} \\ \textbf{upwind choice} : \begin{cases} z_K \textbf{ if } \Phi_{KL}(\boldsymbol{u},\lambda,p)\geq 0, \\ z_L \textbf{ otherwise} \end{cases} \end{cases}$$

Nonlinear convection term: vector divergence

$$\mathrm{div}_K(\boldsymbol{u},\boldsymbol{u},\lambda,p) = \frac{1}{|K|}\sum_{\sigma\in\mathcal{E}_K,\mathcal{T}_\sigma=\{K,L\}} \Phi_{KL}(\boldsymbol{u},\lambda,p)\boldsymbol{u}_\sigma$$

with $\boldsymbol{u}_\sigma$ centred or upwind.

Essential property :

$$\mathrm{div}_{\mathcal{T}}(1,\boldsymbol{u},\lambda,p) = 0 \Rightarrow \begin{cases} \int_\Omega \boldsymbol{u}(\boldsymbol{x})\cdot \mathrm{div}^{\mathrm{c}}_{\mathcal{T}}(\boldsymbol{u},\boldsymbol{u},\lambda,p)(\boldsymbol{x})\mathrm{d}\boldsymbol{x} = 0 \\ \int_\Omega \boldsymbol{u}(\boldsymbol{x})\cdot \mathrm{div}^{\mathrm{up}}_{\mathcal{T}}(\boldsymbol{u},\boldsymbol{u},\lambda,p)(\boldsymbol{x})\mathrm{d}\boldsymbol{x} \geq 0 \end{cases}$$

Choice of the stabilization parameter λ

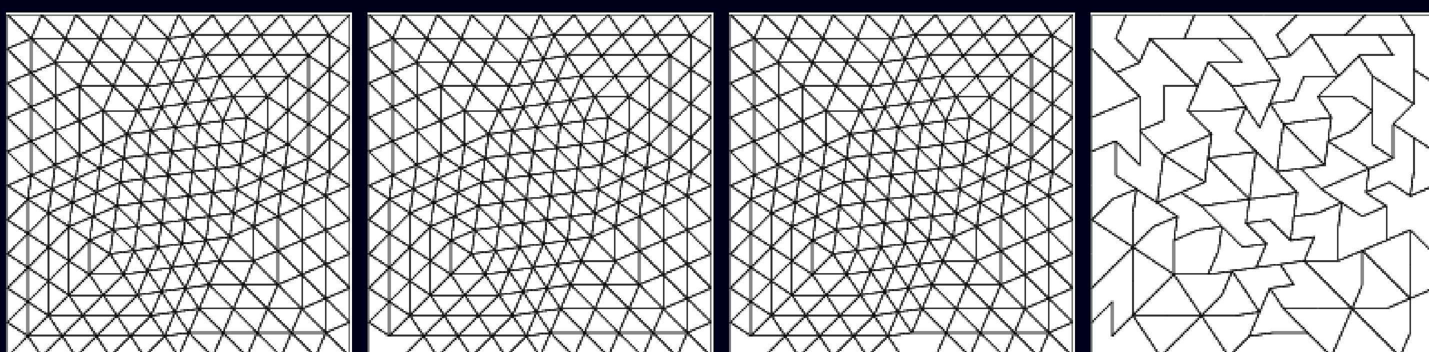
Brezzi-Pitkäranta (easiest choice for the mathematical analysis)

$$\lambda_\sigma = \mathrm{m}(\mathrm{D}_\sigma)\bar{\lambda}\mathrm{h}_{\mathcal{T}}^{\alpha} \; (D_\sigma = D_{K,\sigma} \cup D_{L,\sigma})$$

Stabilization by clusters (more efficient choice in practice)

1. partition $\mathcal{C}$ of the mesh
2. $\lambda_\sigma = \mathrm{m}(\mathrm{D}_\sigma)\bar{\lambda}$ if $\exists\, G \in \mathcal{C}$ with $\mathcal{T}_\sigma \subset G$,
   $\lambda_\sigma = 0$ otherwise.

Example of mesh clustering



Approximation of the pressure gradient

$$\int_\Omega \nabla p \cdot \boldsymbol{v} = - \int_\Omega p \,\mathsf{div} \boldsymbol{v}$$

$$\int_\Omega \widehat{\nabla}_{\mathcal{T}} p \cdot \boldsymbol{v} = - \int_\Omega p \,\mathsf{div}^0_{\mathcal{T}}(\boldsymbol{v}), \ \forall \boldsymbol{v} \in H_{\mathcal{T}}(\Omega)^d$$

with : $\mathrm{div}^0_{\mathcal{T}}(u) = \frac{1}{|K|} \sum_{\sigma \in \mathcal{E}_K, \mathcal{T}_\sigma = \{K,L\}} |\sigma| \Pi_\sigma u \cdot n_{K,\sigma}$

(discrete gradient operator is not necessarily consistent in the finite difference sense)

If $\mathrm{div}_{\mathcal{T}}(1, u, \lambda, p) = 0$ **then** $- \int_\Omega p(x) \mathrm{div}^0_{\mathcal{T}}(u) = \sum_{\sigma \in \mathcal{E}_{\mathrm{int}}, \sigma = K|L} \lambda_\sigma (p_K - p_L)^2$

Complete scheme for steady state Navier-Stokes

$$\int_\Omega \Big(\nu \nabla_{\mathcal{T}} \boldsymbol{u} : \nabla_{\mathcal{T}} \boldsymbol{v} + \mathsf{div}_{\mathcal{T}}(\boldsymbol{u}, \boldsymbol{u}, \lambda, p) \cdot \boldsymbol{v} - p \; \mathsf{div}^{\mathsf{0}}_{\mathcal{T}}(\boldsymbol{v}) \Big) \, \mathsf{d}\boldsymbol{x}$$

$$= \int_\Omega f \cdot \boldsymbol{v} \mathsf{d}\boldsymbol{x}, \; \forall \boldsymbol{v} \in H_{\mathcal{T}}(\Omega)^d$$

$$\mathsf{div}_{\mathcal{T}}(1, \boldsymbol{u}, \lambda, p) = 0$$

Finite volume for convective terms and “sort of finite volume” scheme for diffusive terms

Convergence analysis: estimates

1/ Estimate : $v = u \Rightarrow \displaystyle\sum_{\sigma\in\mathcal{E}_{\rm int},\sigma=K|L} \lambda_\sigma (p_K - p_L)^2 \le C$ and $\|\nabla_{\mathcal{T}} u_{\mathcal{T}}\|_{(L^2(\Omega)^d)^d} \le C$,

so there exists at least one solution.

There exists a subsequence of discretisations and $\bar{u} \in H_0^1(\Omega)^d$ such that

$u_{\mathcal{T}} \longrightarrow \bar{u}$ in $L^2(\Omega)^d$ and $\nabla_{\mathcal{T}} u_{\mathcal{T}} \longrightarrow \nabla \bar{u}$ weakly in $(L^2(\Omega)^d)^d$

2/ $L^2(\Omega)$ estimate on the pressure using Nečas' lemma

$\rightsquigarrow \exists$ subsequence of discretisations and $\bar{p} \in L^2(\Omega)$ with $\displaystyle\int_\Omega \bar{p}(x)\mathrm{d}x = 0$ such that

$p_{\mathcal{T}} \longrightarrow \bar{p}$ weakly in $L^2(\Omega)$

Is $\bar{u}, \bar{p}$ a weak solution to NS ?

Passage to the limit in the scheme : as $h_{\mathcal{T}} \longrightarrow 0$:

$$\nu \int_\Omega \nabla_{\mathcal{T}} u : \nabla_{\mathcal{T}} P_{\mathcal{T}} \varphi \mathrm{d}x \rightarrow \nu \int_\Omega \nabla \bar{u} : \nabla \varphi \mathrm{d}x$$

$$-\int_\Omega p \mathrm{div}^0_{\mathcal{T}}(1, P_{\mathcal{T}} \varphi) \mathrm{d}x \rightarrow -\int_\Omega \bar{p} \mathrm{div} \varphi \mathrm{d}x$$

$$\int_\Omega \mathrm{div}_{\mathcal{T}}(u, u, \lambda, p) \cdot P_{\mathcal{T}} \varphi \mathrm{d}x \rightarrow \int_\Omega \mathrm{div}(\bar{u} \otimes \bar{u}) \cdot \varphi \mathrm{d}x$$

$$\int_\Omega \mathrm{div}_{\mathcal{T}}(1, u, \lambda, p) P_{\mathcal{T}} \varphi \mathrm{d}x \rightarrow \int_\Omega \mathrm{div} \bar{u} \varphi \mathrm{d}x$$

Numerical results

Analytical solution for the Stokes problem, super-admissible meshes (triangles in 2D),

order 2 for velocity in L^2-norm

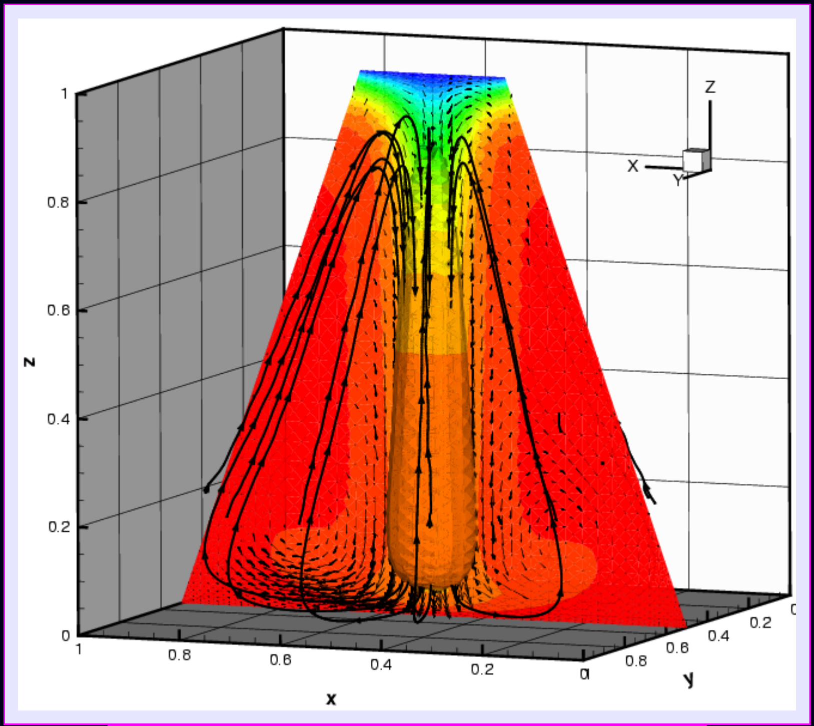
ordre $\frac{3}{2}$ for velocity in a discrete H^1-norm

order $\frac{3}{2}$ for pressure in L^2-norm

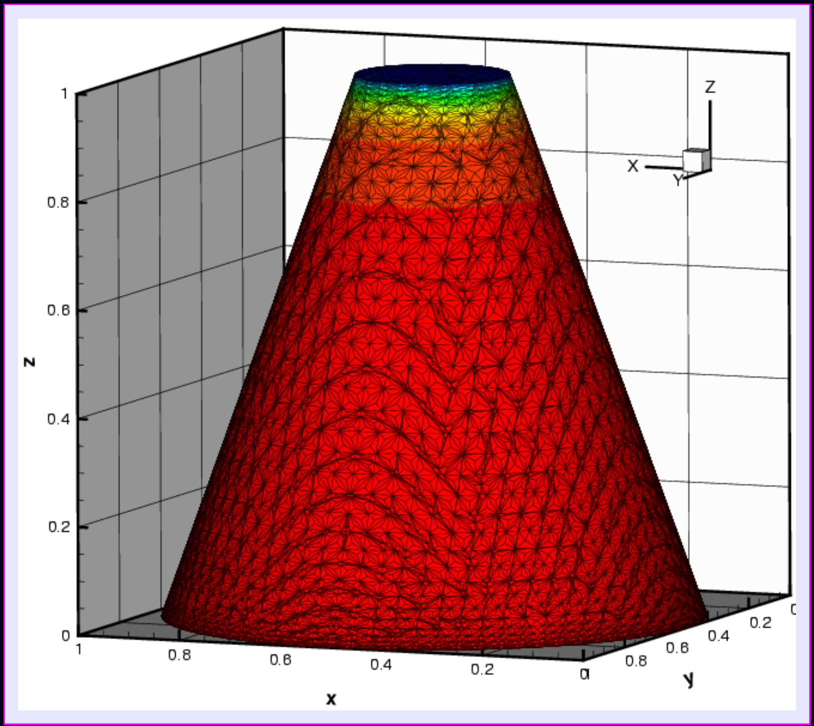
(Proved error estimates for velocity and pressure: order 1 in he L^2-norm)

3D code by E. Chénier Natural convection in a cone heated by below

Adiabatic boundary condition on side walls. Cartesian 3D mesh $40\times40\times40$, truncated by the cone. Locally refined at the top of the cone. $\mathrm{Ra} = 10^6$ and $\mathrm{Pr} = 0.71$



Temperature and velocity in a cross section



Temperature at the wall

Conclusions

Convergent scheme

Efficient scheme for general meshes, allowing complex 3D geometries

SUCCES implemented at IFP for oil reservoir simulations and at IRSN in the ISIS code (combustion)

Perspectives

Compressible Navier-Stokes equations

Generalized MAC scheme

Thanks

Collaboration with

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