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Corrigendum

Corrigendum to "Volume penalization for inhomogeneous Neumann boundary conditions modeling scalar flux in complicated geometry" [J. Comput. Phys. 390 (2019) 452–469]

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This corrigendum contains corrections to some figures and the corresponding text in the article "Volume penalization for inhomogeneous Neumann boundary conditions modeling scalar flux in complicated geometry" [J. Comput. Phys. 390 (2019) 452–469] [1] due to some errors in the implementation of our code. The corrected results yield improved convergence rates and the corrected codes are now available on GitHub [2].

The authors regret that in Ref. [1] the convergence results in Figs. 8, A.16, and C.19 are incorrect, due to bugs in our implementation. In the codes we used to compute the results shown in Figs. 8 and A.16, the boundary values $v(0)$ and $v(\pi)$ were missing in the numerical computation of the integral $\int_0^\pi v(x)\,dx$ using the trapezoidal rule. This has been corrected. Concerning Fig. C.19, the correct setting of the values of the mask function at the interface between solid and fluid regions, $\chi(0) = \chi(\pi) = 1/2$, was likewise missing in our implementation. We now impose $\chi(0) = \chi(\pi) = 1/2$ at the interface. The correct accuracy is second order in h and the errors decay with $O(h^2)$, where h is the grid width. It can be noted that we have $v(0) = -v(\pi)$ due to asymmetry of $v(x)$ for the case where we impose the same values of inhomogeneous Neumann boundary conditions at $x = 0$ and π in the penalized Poisson equation. Similar bugs were in the code for computing the results in Fig. 12 of Ref. [1]. The corrected sentences and figures of the paper are presented below. The results of the rest of the paper remain unaffected. The corrected results showing second order convergence in the one-dimensional (1D) case are in agreement with Ref. [3]. The authors would like to apologize for any inconvenience caused.

The corrections in the text and the new figures are given in the following:
(1) The fourth, fifth and sixth sentences in the second paragraph on page 459 in Section 2.3 and Fig. 8 are modified:

For smaller $\eta\,(= 10^{-3}, 10^{-8})$, we observe that the errors exhibit second order convergence in terms of h, as it is the case in Section 2.2, where identical values of the inhomogeneous boundary conditions are considered on the left and the right boundary (see also Appendix A). In this work, we discretize the term $\beta\, d_x \chi$ at $x = x_i$ by $\beta_i(\chi_{i+1} - \chi_{i-1})/(2h) + O(h^2)$,

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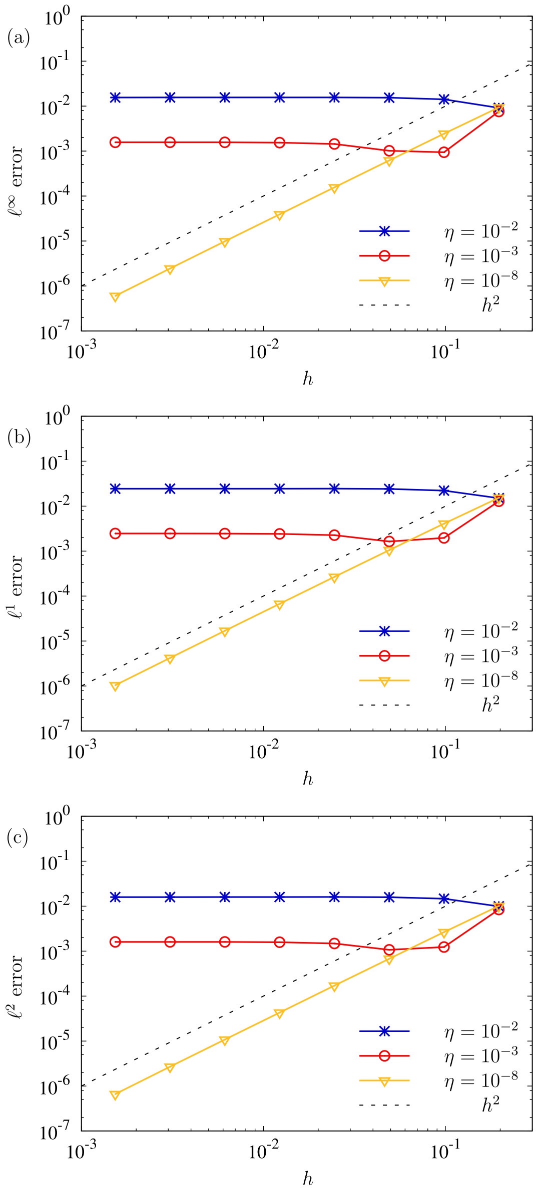

Fig. 8. The errors between the numerical solution of Eq. (22) and the exact solution (21) vs. h for $m = 1$ and $\alpha = 1$ with $\eta = 10^{-2}, 10^{-3}$ and 10^{-8}. The errors are computed in Ω_f, and measured by (a) the ℓ^∞-norm, (b) the ℓ^1-norm and (c) the ℓ^2-norm. The dashed lines show the $O(h^2)$ decay.

which implies that we discretize the delta function at the boundary. Here $\beta_i = \beta(x_i)$ and $\chi_i = \chi(x_i)$. We use the identity $\nabla\cdot(\chi\boldsymbol{\beta}) - \chi\nabla\cdot\boldsymbol{\beta} = \boldsymbol{\beta}\cdot\nabla\chi$. The integral $\int_0^{2\pi}(1-\chi)v\,dx$ is computed with the trapezoidal rule.

(2) The fifth, sixth and seventh sentences in the second paragraph on p. 462 in Section 3 and Fig. 12 are modified:

Fig. 12 shows the dependence of the errors in Ω_f between the numerical solution v and the exact one w as a function of the grid width h. Using least square fitting, we find that the ℓ^∞, ℓ^2 and ℓ^1 errors decay approximately as $h^{1.34}$, $h^{1.62}$ and $h^{1.69}$, respectively. Note that for computing the ℓ^1 and ℓ^2 errors in 2D, we respectively use $(1/N^2)\sum_{\boldsymbol{x}_{ij}\in\Omega_f}|v(\boldsymbol{x}_{ij}) - w(\boldsymbol{x}_{ij})|$ and $(1/N)\sqrt{\sum_{\boldsymbol{x}_{ij}\in\Omega_f}\{v(\boldsymbol{x}_{ij}) - w(\boldsymbol{x}_{ij})\}^2}$. The convergence rate is thus weaker than the second order convergence rate $O(h^2)$, which we observed before in the 1D case. This can be explained by a geometrical error, as the grid points are not aligned with the circular interface. Ref. [4] reported for the penalized 2D Poisson equation with homogeneous Neumann boundary condition at a circular interface likewise no $O(h^2)$ convergence and showed only $O(h)$ convergence in the ℓ^∞ norm. Second order convergence could be achieved for the volume penalization of Neumann boundary conditions, by using interpolation of the mask function as proposed, e.g. in Ref. [5] for the volume penalization of Dirichlet boundary conditions.

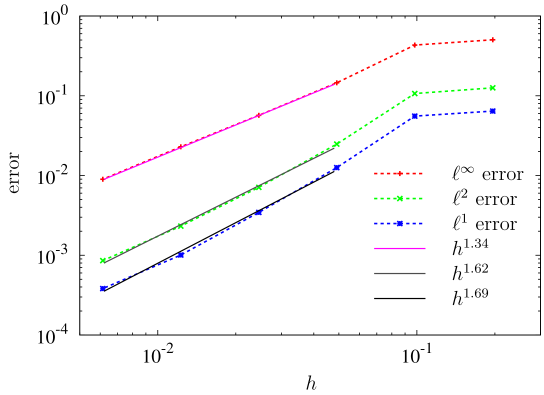

Fig. 12. The h-dependence of the ℓ^∞, ℓ^2 and ℓ^1 errors between the numerical solutions of Eq. (35) and the exact solution (33) for $\alpha = 1$ and $\eta = 10^{-8}$. The errors are computed in Ω_f. The pink, gray and black solid lines respectively show the corresponding $h^{1.34}$, $h^{1.62}$ and $h^{1.69}$ decay obtained by least square fits using the data at $h = 2\pi/N$ ($N = 128, 256, 512, 1024$). (For interpretation of the colors in the figure, the reader is referred to the web version of this article.)

We used successive over-relaxation (SOR) to obtain our numerical solution v of Eq. (35), imposing the condition that $\int_0^{2\pi}\int_0^{2\pi}(1-\chi)v(x,y)dxdy = 0$ which is discretized with the trapezoidal rule. The SOR iteration is stopped when the relative residual error measured in the ℓ^2-norm is below 10^{-9}. We confirmed that the solution v is unchanged, if the criterion 10^{-10} is used.

(3) From the sixth sentence in Section 5 (Conclusion) on page 465 the text should read:

In the case that the same values of the inhomogeneous Neumann boundary conditions are imposed at two interfaces between fluid and solid domains, the errors between the exact solution of the non-penalized equation and the numerical solutions of the discretized equation exhibit second order convergence in terms of the grid width h for sufficiently small η. The eigenvalues of the discretized Laplace operator are identical for homogeneous and inhomogeneous boundary conditions and were studied in Ref. [4]. We also showed that the VP representation needs a source term in the solid domain for the Poisson equation imposing two different values of inhomogeneous Neumann boundary conditions. This term allows for the convergence of the penalized solution to the non-penalized one in the limit of $\eta \to 0$. The numerical solutions of the penalized equation using centered second order finite differences exhibit $O(h^2)$ convergence. Our results suggest that the value of η in the range of $10^{-5} \lesssim \eta \lesssim 10^{-3}$ is optimal for $h \approx 10^{-2}$.

(4) Fig. A.16 is corrected. The last paragraph on page 466 in Appendix A should read:

Fig. A.16 shows that the ℓ^2 error between the numerical solutions v of Eq. (A.4) and the exact one (A.3) for $\eta = 10^{-8}$ using different ϵ. We see the $O(h^2)$ convergence in all cases presented here. The errors measured in the ℓ^1- and ℓ^∞-norms show the similar behavior, and their figures are omitted for brevity.

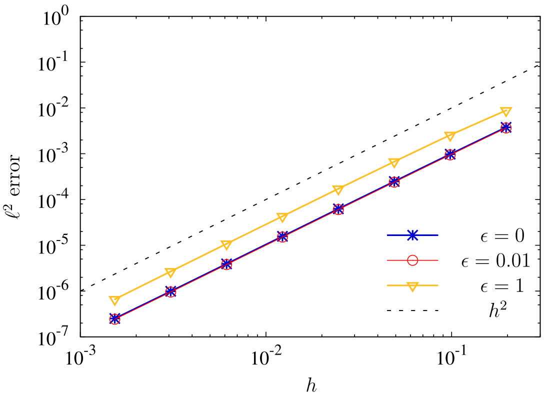

Fig. A.16. The h-dependence of the ℓ^2 error between the numerical solution of Eq. (A.4) and the exact solution (A.3) for $\alpha = 1$ and $\eta = 10^{-8}$ using $\epsilon = 0,\ 0.01$ and 1. The errors are computed in the fluid domain Ω_f.

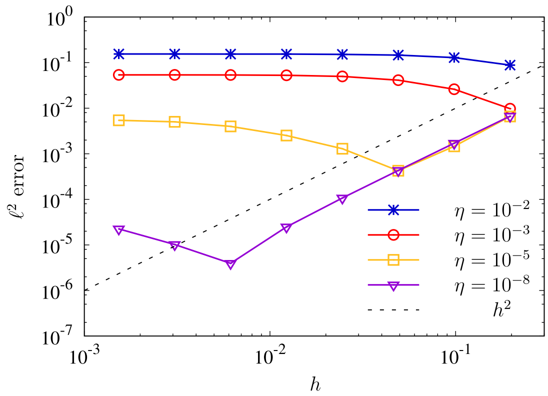

Fig. C.19. The h-dependence of the ℓ^2 errors between the numerical solution of Eq. (C.4) and the exact solution, Eq. (C.3), for $\alpha = 1$ using $\eta = 10^{-2},\ 10^{-3},\ 10^{-5}$, and 10^{-8}. The errors are computed in the fluid domain Ω_f.

(5) Fig. C.19 is corrected. The last paragraph on page 468 in Appendix C should read:

Fig. C.19 shows that the errors between the numerical solutions of Eq. (C.4) and the exact solution w, Eq. (C.3), for $\alpha = 1$ using different $\eta_d = \eta_n = \eta$. For $\eta = 10^{-5}$ and 10^{-8}, the errors decay approximately as $O(h^2)$ with decreasing h, and then saturate due to the penalization error after taking a pronounced minimum, respectively, which corresponds to the optimal value of h for each given η. The optimal values are due to the volume penalization of the Dirichlet boundary conditions as observed in Ref. [6]. The level of the saturation becomes smaller, as η decreases.

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