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NONLINEAR ELLIPTIC EQUATIONS WITH RIGHT HAND SIDE MEASURES

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§ 1 - INTRODUCTION AND STATEMENT OF RESULTS

Let Ω be an open bounded set of $\mathbb{R}^N$ $(N\geq 1)$, $p\in(2-\frac{1}{N}, N]$ and $a : \Omega \times \mathbb{R} \times \mathbb{R}^N \to \mathbb{R}^N$ be a Caratheodory function. We assume that there exist two positive real constants α, β and a function $h(x)\in L^{p'}(\Omega)$ such that for any $s\in\mathbb{R}$, $\xi\in\mathbb{R}^N$, $\eta\in\mathbb{R}^N$ and for almost every $x\in\Omega$

(1) $$a(x,s,\xi)\xi \geq \alpha\, |\xi|^p$$

(2) $$|a(x,s,\xi)| \leq \beta(h(x) + |s|^{p-1} + |\xi|^{p-1})$$

(3) $$[a(x,s,\xi) - a(x,s,\eta)]\, [\xi - \eta]>0 \quad , \quad \xi \neq \eta$$

The first aim of this paper (Theorem 1) is to obtain a solution of the equation

$$(4)\qquad \begin{cases} A(u) = -\mathrm{div}(a(x,u,Du)) = f & \text{in } \Omega \\ u=0 & \text{on } \partial\Omega \end{cases}$$

in the sense of the distributions, when f is a bounded Radon measure on Ω (i.e. $f\in M_b(\Omega)$). To be more precise, we will say that u is a weak solution of (4) if u satisfies

$$(5)\qquad \begin{cases} u\in W_0^{1,1}(\Omega)\ ,\ a(x,u,Du)\in L^1(\Omega) \\ \int_\Omega a(x,u,Du)Dv = \langle f,v\rangle \quad \text{for any} \quad v\in \mathcal{D}(\Omega) \end{cases}$$

We will prove the following existence theorem.

THEOREM 1 - Let $f\in M_b(\Omega)$, then there exists a solution u of (5). Furthermore one has $u\in W_0^{1,q}(\Omega)$ for any $q<\bar{q}=\frac{N(p-1)}{N-1}$.

The theorem above is already known ([BG1]) if a does not depend on u and under a more technical assumption than (3). It is also well known ([LL]) if $p>N$, since in this case $M_b(\Omega)\subset W^{-1,p'}(\Omega)$ and therefore one can use the theory of operators acting between Sobolev spaces in duality.

In (4), we can also consider a lower order term g(x,u) or g(x,u,Du) with a sign condition $g(\cdot,s,\cdot)s\geq 0$, as it is done in [BS], [GM1], [GM2], [BG1], [BG2].

We assume $p\in(2-\frac{1}{N}, N]$ which is equivalent to $\bar{q}>1$. Thus, if $1<p\leq 2-\frac{1}{N}$ Du does not belong to $L^1(\Omega)$ (to overcome this difficulty see the forthcoming paper [X]).

The existence result stated in Theorem 1 is optimal: it is possible to find some $f\in M_b(\Omega)$ (for instance $f=\delta_a$, $a\in\Omega$) or $f\in L^1(\Omega)$ such that the corresponding solution u of (4) does not belong to $W_0^{1,\bar{q}}(\Omega)$.

The existence result in this limiting case is obtained in the following theorem when $|f|\log|f|\in L^1(\Omega)$.

THEOREM 2 - Let $2-\frac{1}{N}<p<N$. Assume that $|f|\log|f|\in L^1(\Omega)$. Then there exists $u\in W_0^{1,\bar{q}}(\Omega)$, $\bar{q}=\frac{(p-1)N}{N-1}$, solution of (5).

A classical method due to G. Stampacchia [S] yields a solution of (4) when A is linear (and p=2), through a duality and a regularity argument in the cases $f\in M_b(\Omega)$ or $|f|\log|f|\in L^1(\Omega)$.

When p=N the weaker assumption $|f|(\log|f|)^{\frac{N-1}{N}}\in L^1(\Omega)$ implies $f\in W^{-1,N'}(\Omega)$ (see [G]) and therefore the existence of a solution $u\in W_0^{1,N}(\Omega)$ follows from [LL].

Finally let $\bar{m}=\frac{Np}{Np-N+p}$ (remark that p=N implies $\bar{m}=1$). If $f\in L^{\bar{m}}(\Omega)$, by Sobolev Imbedding Theorem, f lies in $W^{-1,p'}(\Omega)$ and again the existence of a solution u follows from [LL]. But if $m\in(1,\bar{m})$ and $f\in L^m(\Omega)$, the following theorem can be seen as a regularity theorem regarding the solution u obtained in Theorem 1.

THEOREM 3 - Let $2-\frac{1}{N}<p<N$, $1<m<\bar{m}=\frac{Np}{Np-N+p}$ and $f\in L^m(\Omega)$, then there exists a solution u of (5) which belongs to $W^{1,(p-1)m^*}(\Omega)$.

§ 2 - ESTIMATES.

In order to prove Theorems 1, 2, 3, in this section we will present estimates for u only depending on N, Ω, p, α, $\|f\|_{L^1}$, when f is smooth and u is a solution of (5), given by [LL]. We point out that through this section we will not use assumptions (2), (3).

<u>ESTIMATE 1</u> - Let $f\in L^1(\Omega)\cap W^{-1,p'}(\Omega)$, $1\le q<\bar{q} = \frac{N(p-1)}{N-1}$ and $2-\frac{1}{N}<p\le N$. Then there exists a constant c_1 (depending only on N, α, Ω, p, q, $\|f\|_{L^1}$) such that

$$\int_\Omega |Du|^q \le c_1. \tag{6}$$

<u>ESTIMATE 2</u> - Let $2-\frac{1}{N}<p<N$, $f\in W^{-1,p'}(\Omega)$ and $f\log|f|\in L^1(\Omega)$. Then there exists c_2 (a constant depending only on N, α, Ω, p, $\|f\log|f|\,\|_{L^1}$) such that

$$\int_\Omega |Du|^{\bar{q}} \le c_2 \quad , \quad \bar{q} = \frac{N(p-1)}{N-1} \tag{7}$$

<u>ESTIMATE 3</u> - Let $2-\frac{1}{N}<p<N$, $\bar{m} = \frac{pN}{pN+p-N}$, $1<m<\bar{m}$, and $f\in L^m(\Omega)\cap W^{-1,p'}(\Omega)$. Then there exists a constant c_3 (depending only on N, α, Ω, p, m, $\|f\|_{L^m}$) such that

$$\int_\Omega |Du|^{(p-1)m^*} \le c_3 \tag{8}$$

Estimate 3 stated above improves Proposition 1 of [BG1] where it was proved that $Du\in L^q(\Omega)$, for any $q<(p-1)m^*$.

<u>PROOF OF ESTIMATE 1</u> - The proof is given in (10) of [BG1] (see also Remark 4 of [BG]). In any case, a new proof can be given in the spirit of the following proofs.

In order to prove Estimates 2 and 3, we set (as in [BG1])

$$B_n = \{x\in\Omega : n\le|u(x)| < n+1\}$$

$$A_n = \{x\in\Omega : n\le|u(x)|\} = \bigcup_{k=n}^{\infty} B_k$$

and we define

$$\varphi(s) = \begin{cases} 0 \quad , & 0\le s\le n \\ s-n \quad , & n<s<n+1 \\ 1 \quad , & s\ge n+1 \\ -\varphi(-s) \quad , & s<0 \end{cases} \tag{9}$$

Taking $v=\varphi(u)$ as test function in (4) we have

$$\alpha\int_{B_n} |Du|^p \le \int_{A_n} |f| \tag{10}$$

Our proofs of Estimates 2 and 3 relies on a sharp use of the (right hand side of) inequality (10) because f is assumed to be more "regular" than L^1.

<u>PROOF OF ESTIMATE 2</u> - We use the inequality (10) and we put $\gamma = \frac{\bar{q}}{p}$. Recall that $\bar{q} = \frac{N(p-1)}{N-1}$ and $\bar{q}^* = \frac{\bar{q}}{\bar{q}-p}$. Then

$$(11)\qquad \int_\Omega |Du|^{\bar q} = \int_\Omega \frac{|Du|^{\bar q}}{(1+|u|)^\gamma}\,(1+|u|)^\gamma \le$$

$$\le \left(\int_\Omega \frac{|Du|^p}{(1+|u|)}\right)^{\frac{\bar q}{p}} \left(\int_\Omega (1+|u|)^{\frac{\bar q}{p-\bar q}}\right)^{1-\frac{\bar q}{p}} =$$

$$\le \left(\frac{1}{\alpha}\sum_{n=0}^{\infty} \frac{1}{1+n} \sum_{k=n}^{\infty} \int_{B_k} |f|\right)^{\frac{\bar q}{p}} \left(\int_\Omega (1+|u|)^{\bar q^*}\right)^{1-\frac{\bar q}{p}}$$

$$\le \left(\frac{1}{\alpha}\sum_{k=0}^{\infty} \int_{B_k} |f| \sum_{n=0}^{k} \frac{1}{1+n}\right)^{\frac{\bar q}{p}} \left(\int_\Omega (1+|u|^{\bar q^*}\right)^{1-\frac{\bar q}{p}}$$

$$\le \left(\frac{1}{\alpha}\int_\Omega |f|\left[1+\log(1+|u|)\right]\right)^{\frac{\bar q}{p}} \left(\int_\Omega (1+|u|)^{\bar q^*}\right)^{1-\frac{\bar q}{p}}$$

Now we use the following inequality

$$r\, s \le r \log(1+r) + e^s \qquad \forall r, s \ge 0$$

in order to deduce

$$(12)\qquad \int_\Omega |f|\left[1+\log(1+|u|)\right] \le \int_\Omega |f| + \int_\Omega |f| \log(1+|f|) + \int_\Omega (1+|u|) \le$$

$$\le \int_\Omega |f| \log(1+|f|) + c_4 \;,$$

because we have proved a bound on $\int_\Omega |u|$ in Estimate 1.

Combining (11) and (12) we get

$$(13)\qquad \int_\Omega |u|^{\bar q^*} \le c_5 \left(\int_\Omega |Du|^{\bar q}\right)^{\frac{\bar q^*}{\bar q}} \le c_6 + c_6\left(\int_\Omega |u|^{\bar q^*}\right)^{\frac{p-\bar q}{p}\cdot\frac{\bar q^*}{\bar q}}$$

We remark that $\frac{p-\bar q}{p}\,,\frac{\bar q^*}{\bar q} < 1$ since $p<N$.

Thus we have proved the a priori estimate (7).

<u>PROOF</u> <u>OF</u> <u>ESTIMATE</u> <u>3</u> - Let $q = (p-1)m^*$ $(q<p)$ and $s=q^* \cdot \frac{p-q}{q}$ (>0). Using the inequality (10) as before, we have

$$(14)\qquad \int_\Omega |Du|^q \le \left(\int_\Omega \frac{|Du|^p}{(1+|u|)^s}\right)^{\frac{q}{p}} \left(\int_\Omega (1+|u|)^{\frac{sq}{p-q}}\right)^{1-\frac{q}{p}}$$

$$\le \left(\frac{1}{\alpha}\sum_{n=0}^{\infty} \frac{1}{(1+n)^s} \sum_{k=n}^{\infty} \int_{B_k} |f|\right)^{\frac{q}{p}} \left(\int_\Omega (1+|u|)^{q^*}\right)^{1-\frac{q}{p}}$$

$$\le \left(\frac{1}{\alpha}\sum_{k=0}^{\infty} \int_{B_k} |f| \sum_{n=0}^{k} \frac{1}{(1+n)^s}\right)^{\frac{q}{p}} \left(\int_\Omega (1+|u|)^{q^*}\right)^{1-\frac{q}{p}}$$

We recall that $s<1$ and that

$$\sum_{n=0}^{k} \frac{1}{(1+n)^s} \le c_7\,(1+k^{1-s})\,.$$

Therefore

$$(15)\qquad \sum_{k=0}^{\infty}\int_{B_k} |f| \sum_{n=0}^{k} \frac{1}{(1+n)^s} \le c_7 \int_\Omega |f| + c_7 \int_\Omega |f|\,|u|^{1-s}$$

$$\leq c_7 + c_7 \left(\int_\Omega |f|^m\right)^{\frac{1}{m}} \left(\int_\Omega |u|^{(1-s)m'}\right)^{\frac{1}{m'}}$$

Combining (14) and (15) we have ($q^* = (1-s)m'$)

$$\int_\Omega |Du|^q \leq c_8 + c_8 \left(\int_\Omega |u|^{q^*}\right)^{1-\frac{q}{p}+\frac{q}{pm'}}$$

Then Estimate 3 follows by the previous inequality because $\frac{q}{q^*} > 1-\frac{q}{p} + \frac{q}{pm'}$. Indeed the choice of q is such that $(N-q-Np+Nq)m'=Nq$. That is $[(N-1)m'-N]q=N(p-1)m'$, i.e. $q=(p-1)m^*$. Moreover $q<p$ follows from the inequality $m<\bar{m}$.

<u>REMARK 1</u> - In order to prove the above estimates we have used the test function (9) as in [BG], but it is also possible to use $v=\Phi(u)$, as in [BGV], where

$$\Phi(s) = \int_0^s \frac{dt}{(1+|t|)^s} \cdot$$

Then Estimates 1, 2 and 3 can be proved with (respectively) $s>1$, $s=1$, $s=1-\frac{q^*}{m'}$ ($q=(p-1)m^*$).

§ 3 - PROOFS OF THEOREMS 1, 2, 3.

In this section we take $f \in M_b(\Omega)$ and a sequence $(f_n) \subset W^{-1,p'} \cap L^1(\Omega)$ converging to f and such that $||f_n||_{L^1} \leq ||f||_{M_b}$. We then pass to the limit in the equations

$$(16) \qquad u_n \in W_0^{1,p}(\Omega), \qquad A(u_n) = f_n \quad ,$$

The weak convergence obtained as a consequence of Estimate 1,2 and 3 does not permit to pass to the limit in (16) except when a is linear in Du. A pointwise convergence on Du_n is needed. This is the purpose of Lemma 1 which is also related to some recent result of [BM] and [LM].

<u>PROOF OF THEOREM 1</u> - By Estimate 1, (u_n) is bounded in $W_0^{1,q}(\Omega)$, for any $q<\bar{q}$.

Then we can assume (for some u and for some subsequence still denoted u_n) that

$$(17) \qquad u_n \to u \quad \text{weakly in} \quad W_0^{1,q}(\Omega)$$

$$(18) \qquad u_n \to u \quad \text{strongly in} \quad L^q(\Omega)$$

$$(19) \qquad u_n \to u \quad \text{a.e. in} \quad \Omega \,.$$

This is not sufficient to pass to the limit in (16). We need, for instance $Du_n \to Du$ a.e. This is the content of the following Lemma.

<u>LEMMA 1</u> - Assume (1), (2), (3) and

$$(20) \qquad f_n \text{ bounded in } L^1(\Omega) \quad , \quad f_n \in L^1(\Omega) \cap W^{-1,p'}(\Omega).$$

Then the sequence u_n defined in (16) is compact in $W_0^{1,q}(\Omega)$, for any $q<\bar{q}$.

END OF THE PROOF OF THEOREM 1 - Combining Lemma 1, (2) and (18) we deduce that

$$(21) \qquad a(x,u_n,Du_n) \to a(x,u,Du) \quad \text{in} \quad L^r, \quad \forall r \in s, [\tfrac{N}{N-1})$$

and therefore u is a solution of (4), in the weak sense (5).

PROOF OF THEOREMS 2 AND 3 - The existence results are a consequence of Theorem 1 and Estimate 2 or 3, since

$$a(x,u_n,Du_n) \to a(x,u,Du) \quad \text{weakly in} \quad L^{\frac{N}{N-1}}(\Omega) \quad \text{or in} \quad L^{m^*}(\Omega)$$

REMARK - With our proof of Theorem 3 the convergence of Du_n in $L^{(p-1)m^*}$ is an open problem.

Before proving Lemma 1 we recall that L^1-compactness results for the gradients of a sequence of approximate solutions of nonlinear equations have been obtained in [BMP], [BM], [BG1], [LM], and we emphasize that the first result is contained in a pioneering work by Leray-Lions [LL].

In the proof we will need the following standard

LEMMA 2 - Let (X,T,m) a measurable space, such that $m(X)<\infty$. Let γ be a measurable function, $\gamma : X \to [0,+\infty]$ such that $m(\{x \in X, \gamma(x)=0\})=0$. Then for any $\epsilon>0$, there exists $\delta>0$ such that

$$\int_A \gamma \, dm \leq \delta$$

implies $m(A) \leq \epsilon$.

PROOF OF LEMMA 1 - By Estimate 1, (u_n) is bounded in $W_0^{1,q}(\Omega)$, for any $q<\bar{q}$. Then we can assume (for same $u \in W_0^{1,q}(\Omega)$ and for some subsequence still denoted u_n) that

$$(22) \qquad u_n \to u \quad \text{weakly in} \quad W_0^{1,q}(\Omega)$$

$$(23) \qquad u_n \to u \quad \text{in measure}$$

Our proof relies on the following claim

$$(24) \qquad Du_n \to Du \quad \text{in measure.}$$

In order to prove (24), given $\lambda>0$ and $\epsilon>0$ we set for some $B>1$, $k>0$ $(n,m \in \mathbb{N})$

$$E_1=\{x\in\Omega:|Du_n(x)|>B\}\cup\{x\in\Omega:|Du_m(x)|>B\}\cup\{x\in\Omega:|u_n(x)|>B\}\cup\{x\in\Omega:|u_m(x)|>B\}\,,$$

$$E_2 = \{x\in\Omega : |u_n(x) - u_m(x)| > k\}$$

$$E_3=\{x\in\Omega:|u_n(x)-u_m|\leq k,\ |Du_n(x)|\leq B,\ |Du_m(x)|\leq B,\ |u_n(x)|\leq B,\ |u_m(x)|\leq B,\ |D(u_n-u_m)|\geq\lambda\}.$$

Remark that

$$(25) \qquad \{x\in\Omega : |D(u_n - u_m)(x)| \geq \lambda\} \subset E_1 \cup E_2 \cup E_3\,.$$

Since (u_n) and (Du_n) are bounded in $L^1(\Omega)$, one has meas $E_1 \leq \epsilon$, for B large enough, independently of n, m. Thus we fix B in order to have.

$$\text{meas } E_1 \leq \epsilon .$$

We now take into account meas E_3. Assumption (3) implies that there exists a real valued function $\gamma(x)$ such that

(26) $$\text{meas}(\{x\in\Omega : \gamma(x) = 0\}) = 0$$

and

(27) $$[a(x,s,\xi) - a(x,s,\eta)]\,[\xi - \eta] \geq \gamma(x) \ ,$$

$$\forall\ s\in\mathbf{R},\ \xi,\eta \in \mathbf{R}^N : |s|, |\xi|, |\eta| \leq B \ ,$$

$$|\xi - \eta| \geq \lambda \ , \text{ a.e. } x\in\Omega \ .$$

Indeed there exists a subset C of Ω such that meas(C)=0 and the function $a(x,s,\xi)$ is continuous with respect to (s,ξ) for any $x\in\Omega \setminus C$. Then assumption (3) implies that for $x\in\Omega\setminus C$ and $\xi\neq\eta$ one has

$$[a(x,s,\xi) - a(x,s,\eta)]\,[\xi - \eta] > 0 \ .$$

Define

$$K = \{(s,\xi,\eta) \in \mathbf{R}^{2N+1} : |s| \leq B \ , \ |\xi| \leq B \ , \ |\eta| \leq B \ , \ |\xi - \eta| \geq \lambda\}$$

then

(28) $$\inf\{[a(x,s,\xi) - a(x,s,\eta)]\,[\xi - \eta] : (s,\xi,\eta)\in K\} = \gamma(x) > 0 \ ,$$

since K is compact.

In view of (28)

(29) $$\int_{E_3} \gamma(x) \leq \int_{E_3} [a(x,u_n,Du_n) - a(x,u_n,Du_m)\ D(u_n - u_m) \leq$$

$$\leq \int_{E_3} [a(x,u_m,Du_m) - a(x,u_nDu_m)]\ D(u_n - u_m) +$$

$$+ \int_{E_3} [a(x,u_n,Du_n) - a(x,u_m,Du_m)]\ D(u_n - u_m) \ .$$

If we use $T_k(u_n - u_m)$ in (16) as a test function (where T_k is the usual truncation at levels $\pm$ k) we can say that the last integral is less than or equal to 2kM, where $M\geq||f_n||_{L^1}$. Thus

(30) $$\int_{E_3} \gamma(x) \leq \int_{E_3} [a(x,u_m,Du_m) - a(x,u_n,Du_m)]\ D(u_n - u_m) + 2kM$$

In view of the continuity of $a(x,s,\xi)$ with respect to (s,ξ), for a.e. $x\in\Omega$ and $\overline{\epsilon}>0$ there exists $\delta(x)\geq 0$ (with meas $\{x\in\Omega : \delta(x)=0\}$=0) such that

$$|s\text{-}s'| \leq \delta(x), \ \ |s|, |s'|, |\xi| \leq B \ \text{ imply } \ |a(x,s,\xi) - a(x,s',\xi)| \leq \overline{\epsilon} \ .$$

Remark that $\lim_{k\to 0}$ meas $\{x\in\Omega : \delta(x)<k\}$=0. Let now δ given from Lemma 2 (δ depends on ϵ). We choose $\overline{\epsilon}$ such that $c_9\,\overline{\epsilon} < \delta/3$ and k>0 such that

$$c_9 \int_{E_3\cap\{x:k>\delta(x)\}} [1+h(x)] < \delta/3$$

and

$$2k\ M < \delta/3\ .$$

Then we have

$$\int_{E_3} \gamma(x) < \delta$$

and we can deduce that meas $(E_3) < \epsilon$ independently of n and m.

Now we fix such a k and thanks to the fact that u_n is a Cauchy sequence in measure, we can choose n_0 such that

$$\text{meas } E_2 \leq \epsilon \quad \text{for} \quad n,m \geq n_0\ .$$

Then the convergence (24) and Estimate 1 yield the desired compactness result.

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