Origin of Lagrangian Intermittency in Drift-Wave Turbulence

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The Lagrangian velocity statistics of dissipative drift-wave turbulence are investigated. For large values of the adiabaticity (or small collisionality), the probability density function of the Lagrangian acceleration shows exponential tails, as opposed to the stretched exponential or algebraic tails, generally observed for the highly intermittent acceleration of Navier-Stokes turbulence. This exponential distribution is shown to be a robust feature independent of the Reynolds number. For small adiabaticity, algebraic tails are observed, suggesting the strong influence of point-vortex-like dynamics on the acceleration. A causal connection is found between the shape of the probability density function and the autocorrelation of the norm of the acceleration.

Turbulence is one of the main actors in degrading the confinement quality of magnetically confined fusion plasmas. This so-called micro-turbulence in the edge of plasma fusion devices, such as tokamaks, is commonly admitted to be of electrostatic nature [1,2]. A typical instability leading to this turbulent motion is the drift-wave instability, present in plasmas with a strong magnetic field and a temperature or pressure gradient. Turbulence leads to an enhanced diffusivity and its average influence can be characterized by transport coefficients which represent the mean influence of turbulent motion as an enhanced fluid property [3]. Reviews on the use of transport coefficients in fusion devices are given in [4,5]. Transport coefficients allow us to describe the mean transport on the level of second order moments such as the variance of the impurity density, kinetic energy, and fluxes. The spatial and temporal fluctuations around these variances are however not described by such an approach, since they are directly related to fourth-order moments. These fourth-order moments will give a rough description of the intermittent properties of the turbulence: is the transport bursty, corresponding to non-Gaussian fluctuations or diffusive so that it could be modeled by a Gaussian process? Indeed, if the turbulent transport is dominated by rare but strong events, the impact on the confinement quality will be different from the case where a Gaussian process governs the transport. In three-dimensional fluid turbulence it is now well established that the velocity displays near Gaussian statistics but that the velocity gradients and acceleration are characterized by probability density functions (PDFs) with strongly non-Gaussian tails [6,7]. In two dimensions it was shown that Lagrangian statistics can be strongly non-Gaussian even when the Eulerian statistics are perfectly Gaussian [8]. The present investigation is dedicated to the characterization of Lagrangian intermittency in the close-to-two-dimensional dynamics of electrostatic plasma turbulence.

Intermittency can be investigated through the statistical properties of velocity increments δu, which can be defined both in an Eulerian and in a Lagrangian reference frame. Lagrangian velocity increments are defined as $\delta u(t, \tau) = u(t + \tau) - u(t)$, where $u(t)$ is the Lagrangian velocity, i.e., the velocity of a passive tracer monitored on its trajectory as a function of time. When the shape of the PDF of the velocity increments varies as a function of τ, the statistics are usually said to be intermittent, even though this definition can be criticized [9]. At smallest τ, the PDFs approach the shape of the acceleration PDF, which is generally non-Gaussian in turbulent flows.

The study of the Lagrangian dynamics of fluid turbulence is now possible in controlled turbulence experiments in which small solid tracer particles are followed in the flow (e.g., [10–12]) and numerical simulations of the Navier-Stokes equations [6]. Whereas the experimental tracing of particles in fusion reactors introduces problems related to the extreme conditions in controlled fusion, tracing of particles in numerical simulations of drift-wave turbulence is perfectly possible. In a recent study [9], we presented detailed results on the Lagrangian statistics obtained in simulations of drift-wave turbulence, within the context of the Hasegawa-Wakatani model [13,14]. In the present Letter we will focus on the non-Gaussianity of the acceleration statistics. In particular, we will investigate the influence of the Reynolds number and the collisionality on the statistics and we will propose explanations for the observed behavior.

The Hasegawa-Wakatani model can be derived from the Braginskii two-fluid description [15], considering an ion fluid and an electron fluid in the presence of a fixed magnetic field, assuming isothermal inertialess electrons and cold ions. For details on the derivation of the 2D slab version of Hasegawa-Wakatani equation, we refer, e.g., to [16]. The model assumptions yield eventually a closed set of equations, describing the vorticity $\omega = \nabla^2 \phi$ of the

$E \times B$ motion (with ϕ the electrostatic potential) and the advection of the plasma density fluctuations n:

$$\left(\frac{\partial}{\partial t} - \nu\nabla^2\right)\nabla^2\phi = [\nabla^2\phi, \phi] + c(\phi - n), \qquad (1)$$

$$\left(\frac{\partial}{\partial t} - D\nabla^2\right)n = [n, \phi] - \boldsymbol{u}\cdot\nabla\ln(\langle n\rangle) + c(\phi - n), \quad (2)$$

in which all quantities are suitably normalized as in [17]. The model equations closely resemble the two-dimensional Navier-Stokes equations combined with the advection equation for a scalar n, representing here the fluctuations of the plasma density around a mean profile. Small-scale damping is introduced through the Laplacians, with ν and D denoting viscosity and diffusivity, respectively. Nonlinearities are written as Poisson brackets $[a, b] = \frac{\partial a}{\partial x}\frac{\partial b}{\partial y} - \frac{\partial a}{\partial y}\frac{\partial b}{\partial x}$. The source term in the above equation is the mean plasma-density profile $\langle n\rangle$, which is assumed to be exponentially decaying in the x direction and homogeneous in the y direction, so that Eq. (2) reduces to the advection of a scalar fluctuation with respect to an imposed uniform mean scalar gradient. The electrostatic potential ϕ plays for the $E \times B$ velocity the role of a stream-function, $\boldsymbol{u} = \nabla_\perp\phi$, i.e., $u_x = -\partial\phi/\partial y$ and $u_y = \partial\phi/\partial x$. The Lagrangian acceleration of tracer particles, advected by the $E \times B$ velocity is then

$$\boldsymbol{a}_L = \frac{\partial\nabla_\perp\phi}{\partial t} + [\phi, \nabla_\perp\phi]$$

$$= -\nabla p + \nu\nabla^2\boldsymbol{u} - \frac{\nabla_\perp}{\nabla^2}[c(n - \phi)], \qquad (3)$$

where p is the pressure. The adiabaticity c is given by

$$c = \frac{T_e k_z^2}{e^2 n_0 \eta \omega_{ci}}, \qquad (4)$$

with T_e the electron-temperature, k_z the effective parallel wave number, e the electron charge, n_0 the reference plasma density, η the electron resistivity and ω_{ci} the ion-gyro-frequency. The adiabaticity is therefore determined by the electron resistivity, which is strongest in the edge of fusion devices, where the temperature drops. The proportionality to k_z^2, the square of the dominant wave number in the parallel direction, is a simplification which allows us to reduce the model towards a two-dimensional system. Since the strong magnetic field homogenizes the parallel dynamics, the perpendicular $E \times B$ velocity field is close to two-dimensional. This, in combination with the incompressibility of the $E \times B$ velocity and the assumption that the parallel dynamics is governed by a narrow spectrum peaked around a constant value k_z, allows us to use the above set of equations for the scalars ω and n.

The coupling term $c(\phi - n)$ permits the system to access a saturated turbulent state even in the absence of external forcing. This is the main difference with the equations describing the two-dimensional mixing of a scalar in fluid turbulence. It is related to the presence of the parallel current density, which couples the two equations and gives rise to an electrostatic plasma instability leading to a saturated turbulent state in which the energy is drawn from the imposed mean plasma-density profile. The collisionality of the ions and electrons plays a key role in the model. If the collisionality tends to a large value, hence c becomes small, the equations tend to a hydrodynamic 2D limit in which long-living vortices are observed. It was found in [9] that the Lagrangian acceleration in this case showed a very intermittent behavior, reflected by probability density functions with heavy tails. For intermediate values of c the flow is called quasiadiabatic. The PDFs of the acceleration in this regime tend to exponential distributions.

One remaining open question is whether this intermittent behavior is a Reynolds number effect. Indeed in three-dimensional Navier-Stokes turbulence [12,18] the flatness of the acceleration PDF increases as a function of the Reynolds number for the Reynolds numbers currently available. In the present investigation this Reynolds-number dependence is analyzed by exploiting the results of a set of direct numerical simulations of the Hasegawa-Wakatani model for varying Reynolds number.

Another issue is the relation between the time-correlation of the norm of the acceleration and the manifestation of intermittency as proposed by Mordant *et al.* [19]. The present study will allow us to assess this relation for the different regimes.

Equations (1) and (2) were solved in a double-periodic domain of size 64^2 using a fully dealiased pseudospectral method at a resolution of 1024^2 gridpoints, starting from Gaussian random initial conditions. In the saturated, fully developed turbulent flow 10^4 particles were injected, equally spaced, and their velocity and acceleration were monitored during a large number of large-scale turnover times ($\sim 400 T_e$). The eddy turnover time T_e obtained in the different regimes, defined as $1/\sqrt{\mathcal{W}}$ where $\mathcal{W}$ is the rms vorticity, is of the same order of magnitude, $\sim$0.4. Details on the simulations of Eqs. (1) and (2) can be found in [9] and on the Lagrangian part of the study in [20] in which a similar investigation was performed for Navier-Stokes turbulence. The adiabaticity is varied between $c = 0.01$ and $c = 2$, to obtain different flow regimes. Visualizations of the vorticity field for two flow regimes are shown in Fig. 1.

In Fig. 2, the PDFs of the Lagrangian acceleration are shown for different values of c. It is observed that the PDF evolves from a close to exponential shape for large c to an algebraic shape for $c = 0.01$. To check if this is merely an effect of the Reynolds number, we performed simulations at different Reynolds numbers, which is here defined as $R_\lambda = \lambda\mathcal{U}/\nu$, with $\mathcal{U}$ the rms velocity and $\lambda = \mathcal{U}/\mathcal{W}$, an intrinsic scale of the turbulence. This Reynolds number was varied by a factor 6. The Prandtl number was chosen equal to one for all simulations. The results are shown in Fig. 3, where it is observed that the Reynolds number only slightly influences the shape of the PDFs. Therefore we need to find an alternative explanation for the difference in shapes of the
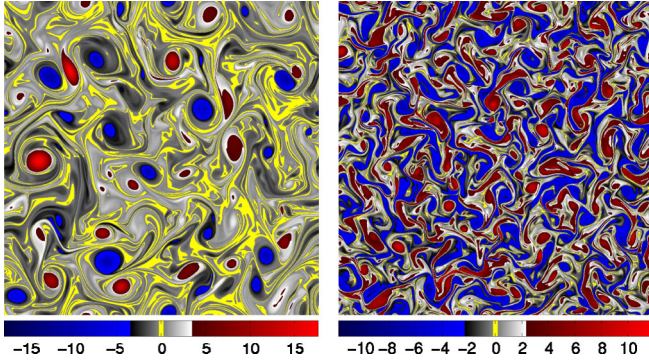
FIG. 1 (color online).   Visualizations of the vorticity field for two different values of the adiabaticity. Left: $c = 0.01$, right: $c = 0.7$.

accelerations for the two different flow regimes. The exponential distributions can be explained as follows: it was argued in [21] that an exponential distribution for the pressure gradient PDF can be obtained from random Gaussian (nonintermittent) velocity fields by simply solving a Poisson-equation to obtain the pressure and subsequently computing the gradient, without considering the nonlinear dynamics of the Navier-Stokes equations. It can be seen from Eq. (3) that the pressure gradient is directly related to the Lagrangian acceleration. The shape of the PDFs for the cases for moderate and large c simply shows that the flow is not intermittent from a Lagrangian point of view, but governed by a Gaussian-like diffusion process.

More puzzling are the algebraic tails, found for small c. In the inset of Fig. 3 we show that the tails show a close to algebraic behavior of the form $p(a) \sim 1/a^\beta$ with β of the order 2. It is interesting to note that the shape of the PDFs obtained in the hydrodynamic case closely resembles the results obtained for point-vortices. Indeed, in Ref. [22] the point-vortex model, introduced by Onsager [23] and Townsend [24], was used to study the influence of point vortices on the Lagrangian acceleration of passive tracers. In their work, the acceleration PDF was to leading order given by $p(a) \sim 1/a^{5/3}$. In this light the results for the quasihydrodynamic flow seem to be at least partially explained by the presence of vortical structures as observed
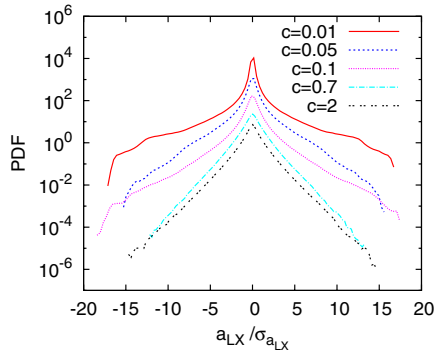
in Fig. 1. The exponent of the power-law tails of the acceleration PDF is close to the value $-5/3$ as in the point-vortex study. Even better agreement might be obtained by comparing with vortex-interaction models using vortices with a finite extension [25] (such as the Burger's vortex).

It remains to be explained why this is not the case for the quasiadiabatic case. As observed in Fig. 1, in this case the drift-waves also seem to organize into vortical structures. However the lifetime of these structures is shorter [26]. The parallel dynamics are thus responsible for the change in lifetime of the vortices. For higher adiabaticity, electrostatic fluctuations are rapidly smoothed out through the parallel current. Vortices do then not exist long enough to influence the acceleration statistics intermittently. In this sense the long-time correlations seem to be essential to obtain the algebraic tails in the acceleration PDF. The centripetal component of the acceleration is constant in a purely circular orbit, and this is captured by the autocorrelation of the norm of the acceleration, which can therefore be directly related to the lifetime of the vortical structures. This is checked in Fig. 4. For all curves, time is normalized by the time at which the autocorrelation of the x component of the acceleration is minimum. This timescale can be qualitatively related to the timescale of the average circular motion of fluid particles. The autocorrelation of the acceleration components displays a behavior similar to what is observed in three-dimensional Navier-Stokes turbulence, with a rapid decrease and a negative dip. This dip becomes less pronounced for lower values of c. It is observed that in the cases in which a closer to exponential decay of the acceleration PDFs is observed, the autocorrelation of the norm decorrelates faster than in the cases in which

FIG. 2 (color online).   PDFs of the Lagrangian acceleration (x component) for different values of c. PDFs are normalized by $\sigma_{a_{LX}}$, the rms value of the acceleration. Graphs for different c are shifted vertically for clarity.
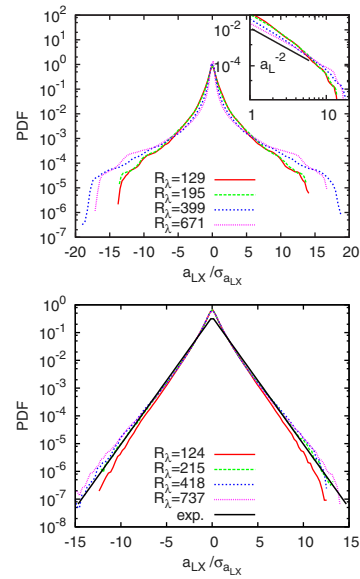

FIG. 3 (color online). PDFs of Lagrangian acceleration at different Reynolds numbers for $c = 0.01$ (top panel) and $c = 0.7$ (bottom panel). In the inset of the top panel, the power-law behavior of the tails of the PDF is demonstrated in double-logarithmic representation.
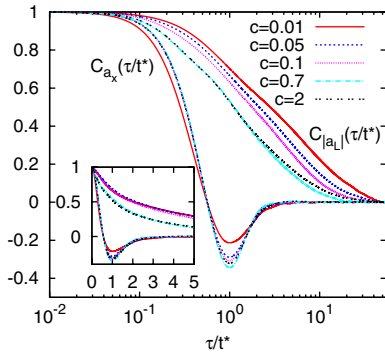
FIG. 4 (color online). Autocorrelations of the acceleration $C_{a_{Lx}}$ and autocorrelation of the norm of the acceleration $C_{|a_L|}$ for the five cases. Inset: Autocorrelations of the acceleration in lin-lin. For each curve, time is normalized by t^*, the time at which the correlation component reaches its minimum value.

the PDFs are algebraically decaying. Indeed the time correlations of the norm become longer for small adiabaticities. This constitutes a proof of the direct relation between the time-correlation of the norm of the acceleration and Lagrangian intermittency as proposed in [19]. A way to numerically check the assumption of the role played by time correlations of the norm of the acceleration within a point-vortex model would be to vary the lifetime of the vortices. If short enough lifetimes are imposed, exponential tails are probably obtained.

The main conclusion of the present work is that the electrostatic turbulence studied here is not intermittent once the adiabaticity is large enough. This corresponds to the case in which the parallel structures have a short enough wavelength (or high parallel wave number) or small collisionality. Intermittency due to electrostatic vortex structures is therefore expected to be stronger near the edge of fusion plasmas, where the collisionality becomes more important.

In the present Letter, the transition between long-living structures and short-lived wavy structures takes place somewhere in between $c = 0.1$ and $c = 0.7$. In reality the parallel spectrum is broadband and we assumed its peak around a certain frequency to obtain the simplified two-dimensional model. If the full three-dimensional model is considered, the dynamics will probably be a mixture between the different cases, dominated by a certain peak-wave number. Also the conclusions of this Letter relate to the dynamics captured within the present model, i.e., homogeneous electrostatic turbulence fed by a strong plasma-density gradient.

For larger adiabaticity ($c > 0.7$), which is expected to correspond to a situation further away from the edge or for colder plasmas, the statistics of this kind of turbulence are close to what would be expected from a Gaussian system. This study suggests that with respect to transport coefficients, microturbulence can be modeled by a Gaussian diffusion process with some additional rare point vortices if the adiabaticity is small enough ($c < 0.7$). This does not imply that plasma turbulence is not intermittent, only that its origin is not due to the mechanism contained in the

present slab geometry if the adiabaticity is large enough. It could be interesting to carry out a similar study in a more complete geometry, such as in the study by Holland *et al.* [27]. In their work, dynamic regimes containing long-living vortices were observed, directly related to the large-scale zonal flows. However, no fully developed turbulent state was considered. Studying the turbulent Lagrangian dynamics in such a geometry constitutes an interesting perspective.

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