Anisotropic Brownian Fields in Mammography

Frédéric RICHARD
richard@math-info.univ-paris5.fr

Université Paris - René Descartes,
MAP5, CNRS UMR 8145
The mammographic image of the female breast is characterized by:
- dense areas occupied by epithelial and stromal tissue,
- translucent zones occupied by fat.

Breast densities vary from one woman to another.
Density and cancer

- Variations are related to the risk of developing breast cancer.
  - Early works of Wolfe (four parenchymal patterns).
  - Classification Bi-Rads of the ACR.
- Mammographic breast densities are one of the strongest breast cancer risk factors.
Breast texture models

- Related to characterization of the mammogram texture (1/f) noise model (Heine 1999, Heine 2002, Burgess 2001).

- An assumption: isotropy of the texture.

- Question:
  - Is isotropy a reasonable assumption?
  - Would a measure of anisotropy help for assessing the cancer risk?
  - How to measure anisotropy?
A random Field: \((X(y), y \in \mathbb{R}^d)\), where \(X(y)\) is a random variable.

Some definitions:
- Centered: \(E(X(y)) = 0\).
- Nul at origin: \(X(0) \equiv 0\).
- Isotropic: \(RX \equiv X\) for any rotation \(R\).
- Self-Similar of order \(\alpha\)
  \[
  X(\lambda \cdot) \equiv \lambda^\alpha X(\cdot).
  \]
- with stationnary increments
  \[
  X(\cdot + t_0) - X(\cdot) \equiv X(t_0) - X(0).
  \]
Fractional Brownian Fields

$H \in (0, 1)$. FBF $B_H(y)$ is the unique random field which
is centered and nul at origin,
gaussian with stationary increments,
$H$ self-similar and isotropic.

Covariance of the FBF:

$\forall x, y, \text{Cov}(B_H(x), B_H(y)) = c_{H,d}(|x|^{2H} + |y|^{2H} - |x-y|^{2H})$.

Spectral representation:

$$\int_{\mathbb{R}^d} \frac{e^{ix\omega} - 1}{|\omega|^{{{1\over 2}(2H+d)}}} dW(\omega), x \in \mathbb{R}^d.$$. 
Hurst Index

The variogram of $B_H$

\[ v(x) = \frac{1}{4} E((B_H(x) - B_H(0))^2) = c_{H,d} |x|^{2H} \]

can characterize the FBF.

$H$ is the Hurst index. It is related to
- the fractal dimension,
- the global regularity of the field,
- the order of self-similarity.

$H$ is a fundamental parameter. There are many methods to estimate it!
A generalization of FBF

(Bonami and Estrade, 2003). Gaussian random fields with stationary increments characterized by a variogram of the form:

\[ v(x) = \int_{\mathbb{R}^d} \sin^2(x \cdot \omega) f(\omega) d\omega. \]

where \( f(\omega) \) is a positive function verifying integrability conditions (called the spectral density).

Anisotropic example obtained with:

\[ f(\omega) = |\omega|^{-\frac{1}{2}} (2H(\frac{\omega}{|\omega|}) + d). \]

where \( H \) is a function of the direction \( \frac{\omega}{|\omega|} \).
Simulation examples

(0.7, 0.7)  (0.7, 0.5)  (0.7, 0.2)
In Anisotropic Fractional Brownian Fields (AFBF)
- Directional regularity is characterized by $H$
- called Directional Hurst Index (DHI).

What is exactly directional regularity in this context?
Hölder regularity

- Hölder condition: almost surely
  
  \[ |X(x) - X(y)| \leq A |x - y|^\alpha, \ x, y \in K \]

- Critical Hölder exponent \( \beta \):
  
  - \( \alpha \in (0, \beta) \), Hölder condition is checked.
  - \( \alpha \in (\beta, 1) \), Hölder condition is not checked.
DHI and line regularity

- Take a direction $u$ of $\mathbb{R}^d$,
- Define $Y_u : t \rightarrow X(tu)$ (line random process in the direction $u$).
- Critical exponent of $Y_u$:
  - \textbf{Isotropic FBF}:
    \begin{equation}
    \forall u, \beta_u = H.
    \end{equation}
  - \textbf{Anisotropic FBF}:
    \begin{equation}
    \forall u, \beta_u = \min_s H(s).
    \end{equation}
- Consequence:
  - $\beta_u$ does not characterize the directional regularity.
  - it is equal to the global regularity of the field.
DHI and projections

- Take a direction \( u \) of \( \mathbb{R}^d \),
- Project \( X \) in the direction orthogonal to \( u \) :

\[
\forall t \in \mathbb{R}, Z_u(t) = \int_{<u>\perp} X(y + tu) dy.
\]

- Critical exponent of \( Z_u \):
  - Isotropic FBF :
    \[
    \forall u, \beta_u = H + 1/2(d - 1).
    \]
  - Anisotropic FBF :
    \[
    \forall u, \beta_u = H(u) + 1/2(d - 1).
    \]
- Characterization through projections.
Directional Average Method

- Consider an image in the plane $I(n, m)$ as a realization of the AFBF on a grid.

- Project images in a direction of the plane, e.g., in the vertical direction

$$P_v(m) = \sum_n I(n, m).$$

- Analyze the regularity of $P_v(m)$ using some usual Hurst index estimators.
Our work

- Theoretical study of estimator asymptotic properties (Bierné, Richard, 2006).
- Evaluation of estimators on synthetic data (Bierné, Richard, 2006).
- Application to mammograms.
Results

Line regularity estimation (in horizontal and vertical directions)

- A FFDM database (30 patients).
- Results are consistent with those of Heine et al. ($H = 0.34 \pm 0.06$).
Results

Projection regularity estimation
(in horizontal and vertical directions)
Results

Horizontal vs vertical projection regularity

Regularite directionnelle hp1 – hp2
Results

Projection regularity vs global regularity

Regularite global min(h01,h02) – directionnelle max(hp1,hp2)