Anna Talarczyk
University of Warsaw

Occupation time fluctuations of branching particle systems

We consider $(d, \alpha, \beta, \gamma)$ branching particle system, which consists of particles moving in $\mathbb{R}^d$ according to a symmetric α-stable Lévy process and branching with a critical $1+\beta$ branching law with probability generating function

$$G(s) = s + \frac{(1-s)^{1+\beta}}{1+\beta}$$

$0 < \beta \leq 1$. The initial positions of the particles are given by a Poisson random measure with intensity measure $\mu_\gamma(dx) = dx/(1+|x|^\gamma)$, $\gamma \geq 0$. The system is described by its empirical process N, where $N_t(A)$ is the number of particles in set A at time t. We investigate the occupation time fluctuations of the system as the time is accelerated, i.e. we are interested in the limit of the processes

$$X_T(t) = \frac{1}{F_T}\left(\int_0^{Tt} N_s ds - E\int_0^{Tt} N_s ds\right), \quad t \geq 0.$$

as $T \to \infty$, where F_T is a proper norming. In some cases also the density of the system is increased. Depending on the interplay between the parameters d, α, β, γ of the system, we obtain several interesting types of limits of normalized occupation time fluctuations. In particular, in "low dimensions" the limits have simple spatial structure (Lebesgue measure) and complicated temporal structure (dependent increments), in "large dimensions" the temporal structure is simple (independent increments) but the spatial structure is more complicated. This is an extension of our previous results, where we studied the system starting from the Poisson random measure whose intensity measure was either Lebesgue ($\gamma = 0$) or finite measure ($\gamma > d$). The limit processes were quite different in these two cases, also the critical dimensions were different. Introducing the measures μ_γ in a way interpolates between these two extreme cases.

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