Approximation of epidemic models by diffusion processes and their statistical inferences

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21/11/2015 CIMPA 2015

TALK 3-Part 3, CIMPA 2015

Recap on MLE asymptotics

MLE asymptotics

θ_0: True value of the parameter.
Θ: Parameter set included in $\mathbb{R}^q$; $\theta_0 \in Int(\Theta)$ $\ell_n(\theta)$: log-likelihood of the parameter θ given the first n observations
$\nabla_\theta g(\theta) = \frac{\partial g}{\partial \theta_i}$ • Rely on tree basic results

- a law of large numbers for the log-likelihood $\ell_n(\theta)$,
- a central limit theorem for the score function
- a law of large numbers for the observed information.

More precisely,
(i) For all $\theta \in \Theta$, $n^{-1}\ell_n(\theta) \to J(\theta_0, \theta)$ $P_{\theta_0}-$ a.s. uniformly w.r. t. Θ,
$\theta \to J(\theta_0, \theta)$ continuous function with a global unique maximum at θ_0 .

(ii) $n^{-1/2}\nabla_\theta \ell_n(\theta_0) \to \mathcal{N}(0, \mathcal{I}(\theta_0))$ in distribution under P_{θ_0} ,
where $\mathcal{I}(\theta))$: Fisher information matrix at θ.

(iii) $lim_{n\to\infty} sup_{|\theta-\theta_0|\leq\delta} \parallel -\frac{1}{n}\nabla^2_\theta \ell_n(\theta) - \mathcal{I}(\theta_0) \parallel \to 0$ as $\delta \to 0$ $P_{\theta_0}-$ a.s.