Do it yourself lookdown constructions: It is safe to build them at home

1. Modeling with generators
2. Sums of generators are generators
3. Hidden variables
4. Modeling deaths
5. Modeling births
6. Event based models

Etheridge and Kurtz (2015)

How to specify a Markov model

An E-valued process is *Markov* wrt $\{\mathcal{F}_t\}$ if

$$E[f(X(t+s))|\mathcal{F}_t] = E[f(X(t+s))|X(t)], \quad f \in B(E)$$

Ordinary differential equations: $\dot{X} = F(X)$

$$X(t+\Delta t) \approx X(t) + F(X(t))\Delta t$$

Stochastic differential equations:

$$X(t+\Delta t) \approx X(t) + F(X(t))\Delta t + G(X(t))\Delta W$$

Infinitesimal specification

Deterministic (ode) case:

$$f(X(t+\Delta t)) \approx f(X(t)) + F(X(t)) \cdot \nabla f(X(t))\Delta t$$

$$\begin{aligned} f(X(t+r)) - f(X(t)) &= \sum f(X(t_{i+1})) - f(X(t_i)) \\ &\approx \sum F(X(t_i) \cdot \nabla f(X(t_i))(t_{i+1} - t_i) \end{aligned}$$

which suggests

$$f(X(t+r)) - f(X(t)) - \int_t^{t+r} F(X(s)) \cdot \nabla f(X(s))ds = 0$$

Martingale properties

"Infinitesimal changes of distribution"

$$E[f(X(t+\Delta t))|\mathcal{F}_t] \approx f(X(t)) + Af(X(t))\Delta t$$

or

$$E[f(X(t+\Delta t)) - f(X(t)) - Af(X(t))\Delta t|\mathcal{F}_t] \approx 0$$

which suggests

$$E[f(X(t+r)) - f(X(t)) - \int_t^{t+r} Af(X(s))ds|\mathcal{F}_t] = 0$$

$$f(X(t)) - f(X(0)) - \int_0^t Af(X(s))ds \quad \text{a martingale}$$

Examples of generators: Jump processes

Poisson process ($E = \{0, 1, 2 \ldots\}$, $\mathcal{D}(A) = B(E)$)

$$Af(k) = \lambda(f(k+1) - f(k))$$

Markov chain (E discrete, $\mathcal{D}(A) = \{f \in B(E) : f \text{ has finite support}\}$)

$$Af(k) = \sum_l q_{k,l}(f(l) - f(k))$$

Pure jump process (E arbitrary)

$$Af(x) = \lambda(x) \int_E (f(y) - f(x))\mu(x, dy)$$

Lévy process

$$Af(x) = \int_{\mathbb{R}^d} (f(x+z) - f(x) - \mathbf{1}_{\{|z| \leq 1\}} z \cdot \nabla f(x))\nu(dx)$$

Examples of generators: Continuous processes

Standard Brownian motion ($E = \mathbb{R}^d$)

$$Af = \frac{1}{2}\Delta f, \quad \mathcal{D}(A) = C_c^2(\mathbb{R}^d)$$

Diffusion process ($E = \mathbb{R}^d$, $\mathcal{D}(A) = C_c^2(\mathbb{R}^d)$)

$$Af(x) = \frac{1}{2}\sum_{i,j} a_{ij}(x)\frac{\partial^2}{\partial x_i \partial x_j}f(x) + \sum_i b_i(x)\frac{\partial}{\partial x_i}f(x)$$

Reflecting diffusion ($E \subset \mathbb{R}^d$)

$$\mathcal{D}(A) = \{f \in C_c^2(\overline{E}) : \eta(x) \cdot \nabla f(x) = 0, x \in \partial E\}$$

$$Af(x) = \frac{1}{2}\sum_{i,j} a_{ij}(x)\frac{\partial^2}{\partial x_i \partial x_j}f(x) + \sum_i b_i(x)\frac{\partial}{\partial x_i}f(x)$$

The martingale problem for A

X is a solution for the martingale problem for (A, ν_0), $\nu_0 \in \mathcal{P}(E)$, if $PX(0)^{-1} = \nu_0$ and

$$f(X(t)) - f(X(0)) - \int_0^t Af(X(s))ds$$

is an $\{\mathcal{F}_t^X\}$-martingale for all $f \in \mathcal{D}(A)$.

Theorem 1 *If any two solutions of the martingale problem for A satisfying $PX_1(0)^{-1} = PX_2(0)^{-1}$ also satisfy $PX_1(t)^{-1} = PX_2(t)^{-1}$ for all $t \geq 0$, then the f.d.d. of a solution X are uniquely determined by $PX(0)^{-1}$*

If X is a solution of the MGP for A and $Y_a(t) = X(a+t)$, then Y_a is a solution of the MGP for A.

Theorem 2 *If the conclusion of the above theorem holds, then any solution of the martingale problem for A is a Markov process.*

Sums of generators are generators

Suppose A and B are generators Let X_n be a stochastic process such that for $k = 0, 2, 4, \ldots$ X_n evolves as if it has generator A on the time interval $[\frac{k}{n}, \frac{k+1}{n})$ and as if it has generator B on the time interval $[\frac{k+1}{n}, \frac{k+2}{n})$. Then for $f \in \widehat{\mathcal{D}} = \mathcal{D}(A) \cap \mathcal{D}(B)$,

$$f(X_n(t)) - f(X_n(0)) - \int_0^t \left(\frac{1 + (-1)^{[ns]}}{n} Af(X_n(s)) + \frac{1 - (-1)^{[ns]}}{n} Bf(X_n(s)) \right)$$

is a martingale. Letting $n \to \infty$, at least along a subsequence X_n should converge to a process such that

$$f(X(t)) - f(X(0)) - \int_0^t \frac{1}{2}(A + B)f(X(s))ds$$

is a martingale for $f \in \widehat{\mathcal{D}}$. (True, for example, if E is compact, $A, B \subset C(E) \times C(E)$, and $\widehat{\mathcal{D}}$ is dense in $C(E)$.)

Hidden variables

A model X corresponding to a generator $\mathbb{A}$.

"Hidden variables" U influence X but are not observable.

$$E[f(U(t))|\mathcal{F}_t^X] = E[f(U(t))] = \alpha f = \int f(u)\alpha(du)$$

(X, U) corresponds to A

$$f(X(t), U(t)) - f(X(0), U(0)) - \int_0^t Af(X(s), U(s))ds$$

is a $\{\mathcal{F}_t^{X,U}\}$-martingale, so

$$E[f(X(t), U(t))|\mathcal{F}_t^X] - E[f(X(0), U(0))|\mathcal{F}_0^X] - \int_0^t E[Af(X(s), U(s))|\mathcal{F}_s^X]ds$$
$$= \alpha f(X(t)) - \alpha f(X(0)) - \int_0^t \alpha Af(X(s))ds$$

should be a $\{\mathcal{F}_t^X\}$-martingale, that is, $\mathbb{A}\alpha f = \alpha Af$.

Primary goal: High density limits

We want an infinite population limit in which we can identify individuals and their relationships to other individuals, for example, their genealogies.

General principle: Keep U and α as simple as possible.

Population models

Modeling finite or infinite populations in which each individual has a location and/or type in E.

Individuals may move and/or mutate (change type).

Must specify how individuals die and how they give birth, and change type.

Assign each individual to a "level" so that observations of X up to time t give no information about the levels at time t.

Assign levels so that in the pre-limiting model, the levels are iid uniform on $[0, \lambda]$.

Elements in the domain

State of the process $\eta = \sum \delta_{(x_i,u_i)}$, a counting measure on $E \times [0,\lambda]$, and $\overline{\eta} = \sum \delta_{x_i}$.

$$f(\eta) = \prod_{(x,u)\in\eta} g(x,u)$$

$0 \leq g \leq 1$ plus regularity as needed

$$\overline{g}(x) = \lambda^{-1} \int_0^\lambda g(x,u)du$$

so

$$E[f(\eta_t)|\mathcal{F}_t^{\overline{\eta}}] = \prod_{x\in\overline{\eta}_t} \overline{g}(x)$$

High density limits

A high-density limit corresponds to $\lambda \to \infty$ while $\lambda^{-1}\overline{\eta}_t^\lambda(C) \to \Xi(t, C)$.

If $g(x, u) = 1$ for $u \geq u_g$ and, perhaps, $x \notin K_g$, then assuming $\eta_t^\lambda \Rightarrow \eta_t$ in the vague, or perhaps weak, topology, then

$$\prod_{(x,u)\in\eta_t^\lambda} g(x, u) \to \prod_{(x,u)\in\eta_t} g(x, u).$$

Since the original levels were iid uniform $[0, \lambda]$, the limiting η_t must be conditionally Poisson with Cox measure $\Xi(t)$. In particular

$$E[\prod_{(x,u)\in\eta_t} g(x, u)|\Xi(t)] = e^{-\int_E \int_0^\infty (1-g(x,u))du\Xi(t,dx)}.$$

In practice, we want to condition on $\mathcal{F}_t^\Xi$.

Modeling deaths

For each $(x,u) \in \eta$, multiply the u by $\rho > 1$ and kill off all particles with $\rho u \geq \lambda$.

If the u are independent and uniformly distributed on $[0,\lambda]$ and independent of the x, and $\rho u < \lambda$, then $u' = \rho u$ is uniformly distributed on $[0,\lambda]$ and independent of the x.

$$P\{\rho u \geq \lambda\} = P\{u > \lambda\rho^{-1}\} = 1 - \rho^{-1} = \frac{\rho - 1}{\rho}$$

Let $0 \leq g(x,u) \leq 1$ and $g(x,u) = 1$ for $u \geq \lambda$. Set $\overline{g}(x) = \lambda^{-1}\int_0^\lambda g(x,u)du$. Then

$$\lambda^{-1}\int_0^\lambda g(x,\rho u)du = \lambda^{-1}\rho^{-1}\int_0^{\rho\lambda} g(x,v)dv = \rho^{-1}\overline{g}(x) + \rho^{-1}(\rho - 1)$$

$$E[\prod_{(x,u)\in\eta_{t-}} g(x,\rho(x)u))|\overline{\eta}_{t-}] = \prod_{x\in\overline{\eta}_{t-}} (\overline{g}(x)\frac{1}{\rho(x)} + \frac{\rho(x)-1}{\rho(x)})$$

Thinning

$f(\eta) = \prod_{(x,u)\in\eta} g(x,u)$

$$A_{th} f(\eta) = \beta(\overline{\eta}) \int_{\mathbb{U}} (\prod_{(x,u)\in\eta} g(x, u\rho(x,z)) - f(\eta))\mu(\overline{\eta}, dz),$$

for some $\rho(x,z) \geq 1$. Let $p(x,z) = \frac{\rho(x,z)-1}{\rho(x,z)}$. Then

$$\alpha A_{th} f(\overline{\eta}) = \beta(\overline{\eta}) \int_{\mathbb{U}} (\prod_{x\in\overline{\eta}} ((1-p(x,z))\overline{g}(x) + p(x,z)) - \alpha f(\overline{\eta}))\mu(\overline{\eta}, dz),$$

When a thinning event of type z occurs individuals are independently eliminated with (type-dependent) probability $p(x,z)$.

High density limit

For the high density limit, assume

$$\lambda^{-1}\overline{\eta} \to \Xi \text{ implies } \beta_\lambda(\overline{\eta}) \to \beta(\Xi).$$

Then

$$A_{th}f(\eta) = \beta(\Xi)\int_{\mathbb{U}}(\prod_{(x,u)\in\eta} g(x, u\rho(x,z)) - f(\eta))\mu(\Xi, dz),$$

and the projected operator becomes

$$\alpha A_{th}f(\Xi) = \beta(\Xi)\int_{\mathbb{U}}(e^{-\int_E \frac{1}{\rho(x,z)}h(x)\Xi(dx)} - \alpha f(\Xi))\mu(\Xi, dz),$$

where $h(x) = \int_0^\infty (1 - g(x,u))du$ and $\alpha f(\Xi) = e^{-\int_E h(x)\Xi(dx)}$.

Pure death generators

For $d_0(x) \geq 0$,

$$A_{pd}f(\eta) = \sum_{(x,u)\in\eta} f(\eta)d_0(x)u\frac{\partial_u g(x,u)}{g(x,u)}.$$

which says the levels satisfy $\dot{u} = d_0(x)u$

$$\alpha A_{pd}f(\overline{\eta}) = \alpha f(\overline{\eta})\sum_{x\in\overline{\eta}} \frac{1}{\overline{g}(x)}\lambda^{-1}\int_0^\lambda d_0(x)u\partial_u g(x,u)du\,.$$

Since

$$\lambda^{-1}\int_0^\lambda u\partial_u g(x,u)du = \lambda^{-1}u(g(x,u)-1)\big|_0^\lambda - \lambda^{-1}\int_0^\lambda (g(x,u)-1)du = 1-\overline{g}(x),$$

$$\alpha A_{pd}f(\overline{\eta}) = \alpha f(\overline{\eta})\sum_{x\in\overline{\eta}} d_0(x)\left(\frac{1}{\overline{g}(x)}-1\right), \qquad (1)$$

which is the generator of a pure death process.

Modeling births

$$
\begin{aligned}
A_{cb}f(\eta) &= f(\eta)\sum_{(x,u)\in\eta} r(x)\Big[\frac{2}{\lambda}\int_u^\lambda (g(x,v)-1)dv + G_1^\lambda(u)\frac{\partial_u g(x,u)}{g(x,u)}\Big] \\
&= f(\eta)\sum_{(x,u)\in\eta} \Big[\frac{2r(x)(\lambda-u)}{\lambda}\frac{1}{\lambda-u}\int_u^\lambda (g(x,v)-1)dv \\
&\qquad + r(x)G_1^\lambda(u)\frac{\partial_u g(x,u)}{g(x,u)}\Big]
\end{aligned}
$$

For each $x \in \overline{\eta}$, write $\overline{\eta}_x$ for $\overline{\eta}\backslash x$. Then

$$
\begin{aligned}
\alpha A_{cb}f(\eta) = \sum_{x\in\overline{\eta}} r(x)f(\overline{\eta}_x)\Big[\frac{1}{\lambda}\int_0^\lambda g(x,u)\frac{2}{\lambda}\int_u^\lambda (g(x,v)-1)dvdu \\
+\frac{1}{\lambda}\int_0^\lambda G_1^\lambda(u)\partial_u g(x,u)du\Big]
\end{aligned}
$$

Calculations

$$\frac{2}{\lambda^2}\int_0^\lambda g(x,u)\int_u^\lambda g(x,v)dvdu = \left(\frac{1}{\lambda}\int_0^\lambda g(x,u)du\right)^2.$$

$$\frac{1}{\lambda}\int_0^\lambda \left(G_1^\lambda(u)\partial_u g(x,u) - \frac{2(\lambda-u)}{\lambda}g(x,u)\right)du. \tag{2}$$

Take

$$G_1^\lambda(u) = \lambda^{-1}(\lambda-u)^2 - (\lambda-u) = \lambda^{-1}u^2 - u$$

Then integrating by parts, (2) reduces to $-\frac{1}{\lambda}\int_0^\lambda g(x,u)du$ and

$$\alpha A_{cb} f(\overline{\eta}) = \alpha f(\overline{\eta})\sum_{x\in\overline{\eta}} r(x)(\overline{g}(x)-1).$$

A branching process

Kurtz and Rodrigues (2011)

$$
\begin{aligned}
Af(\eta) &= f(\eta)\sum_{(x,u)\in\eta} r(x)\Big[\frac{2}{\lambda}\int_u^\lambda (g(x,v)-1)dv + G_1^\lambda(u)\frac{\partial_u g(x,u)}{g(x,u)}\Big] \\
&\qquad + f(\eta)\sum_{(x,u)} f(\eta)d_0(x)u\frac{\partial_u g(x,u)}{g(x,u)} \\
&= f(\eta)\sum_{(x,u)\in\eta}\Big[\frac{2r(x)}{\lambda}\int_u^\lambda (g(x,v)-1)dv \\
&\qquad\qquad + (r(x)G_1^\lambda(u) + d_0(x)u)\frac{\partial_u g(x,u)}{g(x,u)}\Big]
\end{aligned}
$$

and

$$
\alpha Af(\overline{\eta}) = \alpha f(\overline{\eta})\sum_{x\in\overline{\eta}}(r(x)\left[\overline{g}(x)-1\right] + d_0(x)(\frac{1}{\overline{g}(x)}-1))
$$

Critical levels

Suppose r and d_0 are constants. Then

$$\dot{u} = \lambda^{-1} r u^2 + (d_0 - r) u$$

If $d_0 < r$, then $u_c = \frac{\lambda(r-d_0)}{r}$

If the lowest initial level is below u_c, then the population lives forever. If the lowest initial level is above u_c, then the population dies out.

Discrete birth events

The event is determined by

- The number of new particles k.
- The choice of parent with relative chance $r(x)$.
- The placement of the offspring determined by a transition function $q(x, dy)$ from E to E^k.

Mechanism for lookdown construction

The race to become parent

k points are chosen independently and uniformly on $[0,\lambda]$. These will be the levels of the offspring of the event. v^* denotes the lowest of the chosen levels.

For $(x,u)\in\eta$ with $u>v^*$ and $r(x)>0$, let τ_x be defined by

$$e^{-r(x)\tau_x}=\frac{\lambda-u}{\lambda-v^*}.$$

$\frac{\lambda-u}{\lambda-v^*}$ is uniform $[0,1]$, so τ_x is exponential with parameter $r(x)$.

For $(x,u)\in\eta$ satisfying $u<v^*$ and $r(x)>0$, let τ_x be defined by

$$e^{-r(x)\tau_x}=\frac{u}{v^*}$$

Again, τ_x is exponentially distributed with parameter $r(x)$.

Probability of being the parent

The τ_x are independent. Let (x^*, u^*) be the point in η with $\tau_{x^*} = \min_{(x,u)\in\eta} \tau_x$. Then

$$P\{x^* = x'\} = \frac{r(x')}{\int r(x)\overline{\eta}(dx)}, \quad x' \in \overline{\eta}.$$

The new configuration

Assign types $(y_1, \ldots, y_k)$ with joint distribution $q(x^*, dy)$ uniformly at random to the k new levels and transforming the old levels so that

$$\begin{aligned}\gamma_{k,r,q}\eta = \{(x, \lambda - (\lambda - u)e^{r(x)\tau_{x^*}}) : (x,u) \in \eta, \tau_x > \tau_{x^*}, u > v^*\} \\ \cup\{(x, ue^{r(x)\tau_{x^*}}) : (x,u) \in \eta, \tau_x > \tau^*_x, u < v^*\} \\ \cup\{(y_i, v_i), i = 1, \ldots, k\}.\end{aligned}$$

Notice that the parent has been removed from the population and that if $r(x) = 0$, the point (x, u) is unchanged.

Uniformity of levels

For $(x,u) \in \eta$, $(x,u) \neq (x^*,u^*)$, let $h_r^\lambda(x,u,\eta,v^*)$ denote the new level, that is,

$$\begin{aligned} h_r^\lambda(x,u,\eta,v^*) &= \mathbf{1}_{\{u>v^*\}}(\lambda - (\lambda-u)e^{r(x)\tau_{x^*}}) + \mathbf{1}_{\{u<v^*\}}ue^{r(x)\tau_{x^*}} \\ &= \mathbf{1}_{\{u>v^*\}}(ue^{r(x)\tau_{x^*}} - \lambda(e^{r(x)\tau_{x^*}}-1)) + \mathbf{1}_{\{u<v^*\}}ue^{r(x)\tau_{x^*}}, \end{aligned}$$

and

$$f(\gamma_{k,r,q}\eta) = \prod_{(x,u)\in\eta, u\neq u^*} g(x, h_r^\lambda(x,u,\eta,v^*)) \prod g(y_i,v_i).$$

Lemma 3 *Conditional on* $\{(y_i,v_i)\}$ *and* $\overline{\eta}$, $\{h_r^\lambda(x,u,\eta,v^*) : (x,u)\in\eta, u \neq u^*\}$ *are independent and uniformly distributed on* $[0,\lambda]$.

Limit as λ goes to ∞

Suppose $\lambda \to \infty$, $\lambda^{-1}\overline{\eta} \to \Xi$, and $\lambda^{-1}k \to \zeta$. Then, in the limit:

The new levels in a birth event form a Poisson process with intensity ζ.

v^* will be exponentially distributed with parameter ζ.

$u^* > v^*$ and $\tau^\lambda_{x^*} \to 0$.

For $u > u^*$, $h^\lambda_r(x, u, \eta, v^*) \to u - (u^* - v^*)\frac{r(x)}{r(x^*)}$.

Event based models

cf. Berestycki, Etheridge, and Hutzenthaler (2009)

Etheridge (2000); Barton, Etheridge, and Véber (2010); Véber and Wakolbinger (2015)

A discrete event birth generator will be of the form

$$A_{db} f(\eta) = \int_{\mathbb{U}} (H_z(g,\eta) - f(\eta))\mu(dz),$$

where

$$\begin{aligned} H_z(g,\eta) &= \lambda^{-k(z)} \int_{[0,\lambda]^{k(z)}} \prod_{(x,u)\in\eta, u\neq u^*(\eta,v^*)} g(x, h^{\lambda}_{r(\cdot,z)}(x,u,\eta,v^*)) \\ &\qquad \times \prod_{i=1}^{k(z)} \int_E g(y_i,v_i) q(x^*(\eta,v^*), z, dy_i) dv_1 \ldots dv_{k(z)}. \end{aligned}$$

μ may be σ-finite and has the interpretation that there exists a Poisson random measure ξ with mean measure $\mu \times \ell$ on $\mathbb{U} \times [0,\infty)$, and if $(z,t) \in \xi$, then at time t,

$$E[f(\eta_t)|\eta_{t-}] = H_z(g,\eta_{t-}).$$

Birth event followed by thinning

$$A_{db,th}f(\eta) = \int_{\mathbb{U}} (H_z(g,\eta) - f(\eta))\mu(dz),$$

where

$$H_z(g,\eta) \;=\; \lambda^{-k(z)} \int_{[0,\lambda]^{k(z)}} \prod_{(x,u)\in\eta, u\neq u^*(\eta,v^*)} g(x, \rho(x,z)h^{\lambda}_{r(\cdot,z)}(x,u,\eta,v^*))$$
$$\times \int \prod_{i=1}^{k(z)} \int_E g(y_i, \rho(y_i,z)v_i)q(x^*(\eta,v^*), z, dy_i)dv_1 \ldots dv_{k(z)}$$

Note that (x^*, u^*) is a function of η and v^*, and if and event z occurs at time t, then

$$\eta_t = \sum_{(x,u)\in\eta_{t-}, u\neq u^*} \mathbf{1}_{\{\rho(x,z)h^{\lambda}_{r(\cdot,z)}(x,u,\eta_{t-},v^*)<\lambda\}}\delta_{(x,\rho(x,z)h^{\lambda}_{r(\cdot,z)}(x,u,\eta_{t-},v^*))}$$
$$+ \sum_{i=1}^{k(z)} \mathbf{1}_{\{\rho(y_i,z)v_i<\lambda\}}\delta_{(y_i,\rho(y_i,z)v_i)}$$

Projected model

$$\alpha A_{db,th} f(\overline{\eta}) = \int_{\mathbb{U}} \sum_{x^* \in \overline{\eta}} \frac{r(x^*, z)}{\int r(x,z)\overline{\eta}(dx)} (\overline{H}_z(g, \overline{\eta}, x^*) - \alpha f(\overline{\eta}))\mu(dz)$$

where, setting $p(x,z) = \frac{\rho(x,z)-1}{\rho(x,z)}$ and $\overline{\eta}_{x^*} = \overline{\eta} - \delta_{x^*}$,

$$\begin{aligned} \overline{H}_z(g, \eta, x^*) &= \lambda^{-k(z)} \int_{[0,\lambda]^{k(z)}} \prod_{x \in \overline{\eta}_{x^*}} ((1 - p(x,z))\overline{g}(x) + p(x,z)) \\ &\quad \times \prod_{i=1}^{k(z)} \int_E ((1 - p(y_i, z))\overline{g}(y_i) + p(y_i, z)) q(x^*, z, dy_i). \end{aligned}$$

High density limit

$\mu^\lambda(dz)$ governs the appearance of events of the form

$$(k(z), r(x,z), q(x,z,dy), \rho(x,z))$$

Assume

$$\int_{\mathbb{U}} h(\frac{k(z)}{\lambda})\varphi(z)\mu(dz) \to \int_{\mathbb{U}}\int_0^\infty h(\zeta)\mu_\zeta(d\zeta, z)\varphi(z)\mu(dz).$$

$$
\begin{aligned}
H_z(g,\eta) &= \lambda^{-k(z)} \int_{[0,\lambda]^{k(z)}} \prod_{(x,u)\in\eta, u\neq u^*(\eta,v^*)} g(x,\rho(x,z)h^{\lambda}_{r(\cdot,z)}(x,u,\eta,v^*)) \\
&\quad \times \int \prod_{i=1}^{k(z)} \int_E g(y_i,\rho(y_i,z)v_i)q(x^*,z,dy_i)dv_1\ldots dv_{k(z)} \\
\to \int_0^\infty \int_0^\infty \Big[\zeta e^{-\zeta v^*} &\prod_{(x,u)\in\eta, u\neq u^*(\eta,v^*)} g(x,\rho(x,z)(u-\mathbf{1}_{\{u>u^*\}}(u^*-v^*)\frac{r(x)}{r(x^*)})) \\
&\quad \times \int_E g(y,\rho(y,z)v^*)q(x^*,z,dy) \\
&\quad \times \exp\{-\zeta \int_E \int_{v^*}^\infty (1-g(y,\rho(y,z)v)q(x^*,z,dy)dv)\}\Big] dv^* \mu_\zeta(d\zeta,z)
\end{aligned}
$$

Projected generator

Therefore, $\alpha f(\Xi) = e^{-\int_E h(x)\Xi(dx)}$ and setting

$$p(x^*, \Xi) = \frac{r(x^*, z)}{\int_E r(x, z)\Xi(dx)}$$

$$\mathcal{H}_z(g, \Xi) = \int_0^\infty \int_0^\infty \Big[\exp\{-\int_E \frac{1}{\rho(x, z)} h(x)\Xi(dx)\} \\ \times \int_E p(x^*, \Xi) \exp\{-\zeta \int_E \frac{1}{\rho(y, z)} h(y)q(x^*, z, dy)dv)\}\Xi(dx^*)\Big] \mu_\zeta(d\zeta, z)$$

where $h(x) = \int_0^\infty (1 - g(x, u))du$,

$$\alpha A_{bd,th}^\infty f(\Xi) = \int_{\mathbb{U}} (\mathcal{H}_z(g, \Xi) - \alpha f(\Xi))\mu(dz)$$

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Abstract

Do it yourself lookdown constructions: It is safe to build them at home

"Lookdown" constructions provide representations of population models in terms of countable systems of particles in which each particle has a "type" which may record both spatial location and genetic type and a "level" which incorporates the lookdown structure. At first glance, the constructions may appear very mysterious and difficult to apply. The goal of the talk will be to show how to break the population model of interest into pieces, to show how a lookdown process can be defined for each piece, and then to see that the pieces come together to give a lookdown construction for the full model. The talk is based on a forthcoming paper with Alison Etheridge