Contribution to the study of the convergence of numerical schemes of partial differential equations

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Aim of the presentation

The aim of this presentation is to give a vision on:

1. My career as a Teacher
2. My career as Researcher
3. Some of my works under preparation

Plan of this presentation

1. Curriculum Vitae: Education, Positions, Courses taught, Supervisions, Research projects, Invitation to gives Conferences, and other activities.
2. Axes of research
3. Works under preparation and perspectives

Curriculum Vitae-Education

- December 9th 2009: Algerian HDR.
  - Referees: Professors F. Benkhaldoun and R. Eymard
- November 14th 2005: Ph.D Thesis in Applied Mathematics, Aix Marseille I University–France.
  - Supervisor: Professor T. Gallouët.
  - Referees: Professors Y. Maday and M. Moussaoui.
- July 1996: Magister in Applied Mathematics, Annaba–Algeria.
- July 1994: DEA (Diplôme d'Etudes Approfondies) in Applied Mathematics, Annaba–Algeria.
- July 1993: DES (Diplôme d'Etudes Supérieures) in Functional Analysis, Annaba–Algeria.

Curriculum Vitae-Positions

- Since Jan. 8th 15: Professor at Annaba University–Algeria.
- Dec. 9th 09–Jan. 7th 15: Maître de Conférance–A, at Annaba–Algeria.
- Since Nov. 16th 1999: Enseignant Chercheur at Annaba–Algeria.
- Mar. 06–May 07: Post-Doc in WIAS (Weierstrass Institute for Applied Analysis and Stochastics), Berlin–Germany.
- May 07–Dec. 07: Post-Doc in NCMM (Nečas Center of Mathematical Modeling), Praha–Czech Republic.
- Sep. 04–Aug. 05: ATER, University of Marseille I–France.
- Oct. 93–Oct.99: Lecturer of Mathematics at Annaba-University and Cherchell (Military Service).

Curriculum Vitae-Courses Taught

- Under-graduation (Tronc Commun) and Graduation: Analysis, Statistics, Probability, Algebra, Numerical Analysis, and Applications of Mathematics in other disciplines.
- Post-Graduation (Master I and II): Numerical Analysis for PDEs and Finite Volume methods.

Curriculum Vitae-Some supervisions Master II at Annaba University

- 2013/2014: Ms. “Loujani, Nour El-Houda”: in a subject on “Error Estimates for the Finite Volume Approximation for the Heat Equation”. .
- 2012/2013: Ms. “Guenadil, Assia” and Mr. “Bouabda, Achour”: “Numerical Methods for Schrödinger equation ”.
- 2010/2011: Ms. “Sebti, Habiba”: “Numerical Methods for Fractional derivative Equations”. “Memoire” defended in June 19th 2011.

Curriculum Vitae-Expert for Phd theses

- September, 2014: Phd thesis of Mrs. “A. Assala”. Thesis entitled “Etude Mathématique et Numérique de certains Problèmes des Milieux Poreux”. University of Annaba-Algeria.
- June, 2014: Ph.d thesis of Mrs. “N. Ouanes”. Thesis entitled “Cycles Limites des Systèmes Différentiels de Liénard perturbés”. University of Annaba-Algeria.
- September, 2013: Ph.d thesis of Mr. “M. Kara”. Thesis entitled “Calcul des Modes de Torsion et la Méthode de Domaines Fictifs pour les Problèmes d’Elasticité Plane avec des Conditions aux Limites Générales”. University of Setif-Algeria.

Curriculum Vitae-Chef of Research Projects at Annaba University

- 2015/2019: Chef of the CNEPRU[1] project "Les Méthodes de Volumes et Eléments Finis pour Quelques Modèles en Dynamique de Fluide".
- 2011/2013: Chef of PNR[2] project "Etude Mathématique et numériques de quelques modeles de dynamique de Gaz, exemple systeme des equation d'Euler de la dynamique de Gaz".
- 2011/2014: Chef of the CNEPRU project "L' Analyse mathématique et numérique de la récupération assistée des hydrocarbures" (Mathematical and numerical analysis of enhanced oil recovery).

[1] Comité National d'Evaluation et de Programmation de la Recherche Universitaire
[2] Programmes Nationaux de la Recherche

Curriculum Vitae-Invitation to give conferences

- WIAS (Weirstrass Institute of Applied Analysis and Stochastics):
  - June 2016: TBA.
  - September 13: Some recent results on the convergence order of finite volume methods for evolution equations on general nonconforming multidimensional spatial meshes.
  - February 06: Finite volume methods for elliptic problems.
- Department of Mathematics and Statistics of Memorial University, Newfoundland-Canada. January 08: On some higher order finite volume approximations on lower order schemes and application on finite element method.
- Departamento de Matematica, Instituto Superior Tecnico, Lisboa-Portugal. June 06: An approach to improve convergence order in finite volume and its application in finite element method.

Curriculum Vitae-Other activities

- Reviewer for Zentralblatt MATH: since April 23, 2008.
  - Documents reviewed: more than 148 Reviews
- Reviewer for AMS (Mathematical Society): since March 31, 2008.
  - Documents reviewed: more than 60 Reviews.
- Reviewer or Invited to reviewer in journals): Journal of the Franklin Institute, AMC (Applied Mathematics and Computation).

Axes of research

First axis

Convergence and convergence order in Finite Volume methods.

Second axis

Convergence order in Finite Element methods.

Third axis

High convergence order using Low Order Schemes.

Fourth axis

Convergence order of the COMSOL solutions.

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Introduction: Finite Volume methods from Admissible meshes to SUSHI

Finite volume methods are numerical methods approximating different types of Partial Differential Equations (PDEs). They are based on three principle ideas:

- Subdivision of the spatial domain into subsets called Control Volumes.
- Integration of the equation to be solved over the Control Volumes.
- Approximation of the derivatives appearing after integration.

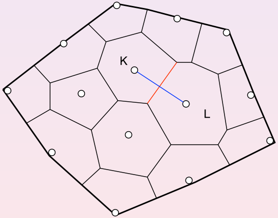
[1] We mean here the "pure" finite volume methods and not finite volume-element methods

Introduction (suite): Finite Volume methods on admissible meshes

Definition

Let $\mathcal{T}$ be an Admissible Mesh in the sense of Eymard et al. (Handbook, 2000).

$K \in \mathcal{T}$ are the control volumes and σ are the edges of the control volumes K.



$T_{K,L} = m_{K,L} / d_{K,L}$

Introduction (suite): Finite Volume methods on admissible meshes

Main properties of Admissible mesh:

1. Convexity of the Control Volumes.
2. The orthogonality property: the $(\boldsymbol{x}_K\boldsymbol{x}_L)$ is orthogonal to the common edge σ between the control volumes K and L.

Introduction (suite): Finite Volume methods on admissible meshes

Model to be solved:

$$-\Delta u(\boldsymbol{x}) = f(\boldsymbol{x}),\ \boldsymbol{x}\in\Omega \quad \text{and} \quad u(\boldsymbol{x}) = 0,\ \boldsymbol{x}\in\partial\Omega. \tag{1}$$

Principles of Finite Volume scheme:

1. Integration on each control volume K : $-\int_K \Delta u(\boldsymbol{x})d\boldsymbol{x} = \int_K f(\boldsymbol{x})d\boldsymbol{x}$,
2. Integration by Parts gives : $-\int_{\partial K} \nabla u(\boldsymbol{x})\cdot \mathbf{n}(\boldsymbol{x})d\gamma(\boldsymbol{x}) = \int_K f(\boldsymbol{x})d\boldsymbol{x}$
3. Summing on the lines of K: $-\sum_{\sigma\in\mathcal{E}_K}\int_\sigma \nabla u(\boldsymbol{x})\cdot \mathbf{n}(\boldsymbol{x})d\gamma(\boldsymbol{x}) = \int_K f(\boldsymbol{x})d\boldsymbol{x}$

Introduction (suite): Finite Volume methods on admissible meshes

Approximate Finite Volume Solution $u_{\mathcal{T}} = (u_K)_K$

$$-\sum_{\sigma \in \mathcal{E}_K} \frac{\mathrm{m}(\sigma)}{d_{K|L}} (u_L - u_K) = \int_K f(\boldsymbol{x}) d\boldsymbol{x}. \qquad (2)$$

Matrix Form

$$\mathcal{A}^{\mathcal{T}} u_{\mathcal{T}} = f_{\mathcal{T}}.$$

Introduction (suite): Finite Volume methods on admissible meshes

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Matrix Form

$$\mathcal{A}^{\mathcal{T}} u_{\mathcal{T}} = f_{\mathcal{T}}.$$

Introduction (suite): Finite Volume methods on admissible meshes

Theorem

Let $\mathcal{X}(\mathcal{T})$: functions which are constant on each control volume K. Let $e_{\mathcal{T}} \in \mathcal{X}(\mathcal{T})$ be defined by $e_K = u(\boldsymbol{x}_K) - u_K$ for any $K \in \mathcal{T}$. Assume that the exact solution u satisfies $u \in \mathcal{C}^2(\overline{\Omega})$. Then the following convergence result hold:

1. H_0^1*-error estimate*

$$\|e_{\mathcal{T}}\|_{1,\mathcal{T}} \leq Ch\|u\|_{2,\overline{\Omega}}, \tag{3}$$

where $\|\cdot\|_{1,\mathcal{T}}$ *is the* H_1^0*-norm* $\|e_{\mathcal{T}}\|_{1,\mathcal{T}}^2 = \sum_{\sigma=K|L\in\mathcal{E}} \frac{\mathrm{m}(\sigma)}{d_\sigma}(u_L - u_K)^2$.

2. L^2*-error estimate:*

$$\|e_{\mathcal{T}}\|_{L^2(\Omega)} \leq Ch\|u\|_{2,\overline{\Omega}}. \tag{4}$$

Introduction (suite): Finite Volume methods using nonconforming grids, SUSHI scheme

Definition (New mesh of Eymard et *al.*, IMAJNA 2010)

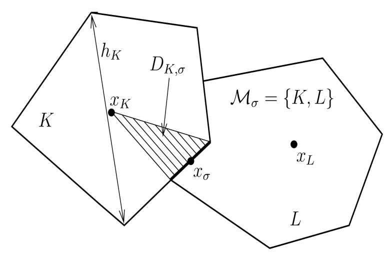

Figure : Notations for two neighbouring control volumes in $d = 2$

Introduction (suite): Finite Volume methods using nonconforming grids, SUSHI scheme

Main properties of this new mesh:

1. (mesh defined at any space dimension): $\Omega \subset \mathbb{R}^d$, $d \in \mathbb{N}$
2. (orthogonality property is not required): the orthogonality property is not required in this new mesh. But, additional discrete unknowns are required.
3. (convexity): the classical admissible mesh should satisfy that the control volumes are convex, whereas the convexity property is not required in this new mesh.

Introduction (suite): Finite Volume methods using nonconforming grids, SUSHI scheme

Principles of discretization for Poisson's equation:

1. Discrete unknowns: the space of solution as well as the space of test functions are in

$$\mathcal{X}_{\mathcal{D},0} = \{\left((v_K)_{K\in\mathcal{M}}\,,\,(v_\sigma)_{\sigma\in\mathcal{E}}\right),\;\; v_K, v_\sigma \in \mathbb{R},\;\; v_\sigma = 0,\, \forall\sigma\in\mathcal{E}_{\rm ext}\}$$

2. Discretization of the gradient: the discretization of ∇ can be performed using a stabilized discrete gradient denoted by $\nabla_{\mathcal{D}}$, see Eymard et *al.* (IMAJNA, 2010):
    1. The discrete gradient $\nabla_{\mathcal{D}}$ is stable
    2. The discrete gradient $\nabla_{\mathcal{D}}$ is consistent.

Introduction (suite): Finite Volume methods using nonconforming grids, SUSHI

Weak formulation for Poisson's equation: Find $u \in H_0^1(\Omega)$ such that

$$\int_\Omega \nabla u(\boldsymbol{x}) \cdot \nabla v(\boldsymbol{x}) d\boldsymbol{x} = \int_\Omega f(\boldsymbol{x}) v(\boldsymbol{x}) d\boldsymbol{x}, \quad \forall v \in H_0^1(\Omega). \tag{5}$$

SUSHI (Scheme Using stabilized Hybrid Interfaces) for Poisson's equation: Find $u_{\mathcal{D}} \in \mathcal{X}_{\mathcal{D},0}$ such that

$$\int_\Omega \nabla_{\mathcal{D}} u_{\mathcal{D}}(\boldsymbol{x}) \cdot \nabla_{\mathcal{D}} v(\boldsymbol{x}) d\boldsymbol{x} = \int_\Omega f(\boldsymbol{x}) v(\boldsymbol{x}) d\boldsymbol{x}, \quad \forall v \in \mathcal{X}_{\mathcal{D},0}. \tag{6}$$

Introduction (suite): Finite Volume methods using nonconforming grids, SUSHI

Theorem

Assume that the exact solution u satisfies $u \in \mathcal{C}^2(\overline{\Omega})$. Then the following convergence result hold:

1. H_0^1*-error estimate*

$$\|\nabla u - \nabla_{\mathcal{D}} u_{\mathcal{D}}\|_{L^2(\Omega)^d} \leq Ch\|u\|_{2,\overline{\Omega}}. \tag{7}$$

2. L^2*-error estimate:*

$$\|u - \Pi_{\mathcal{M}} u_{\mathcal{D}}\|_{L^2(\Omega)} \leq Ch\|u\|_{2,\overline{\Omega}}. \tag{8}$$

First work: Finite volume for oblique boundary value problems.

Subject of this work:

Error estimate for finite volume approximate solutions of some oblique derivative boundary problems. With T. Gallouët. IJFV, 2006.

Model problem

$$-\Delta u(\boldsymbol{x}) = f(\boldsymbol{x}), \quad \boldsymbol{x} \in \Omega \tag{9}$$

where Ω is an open bounded polygonal subset in $\mathbb{R}^2$ and, for a given $\alpha \in \mathbb{R}$

$$u_n(\boldsymbol{x}) + \alpha u_t(\boldsymbol{x}) = 0, \ \boldsymbol{x} \in \partial\Omega, \tag{10}$$

where $u_n = \nabla u \cdot \mathbf{n}$ and $u_t = \nabla u \cdot \mathbf{t}$ with $\mathbf{n} =$ and $\mathbf{t}$ are respectively the normal vector to the boundary $\partial\Omega$ and outward to Ω and tangential derivative.

First contribution (suite): Finite volume for oblique boundary value problems.

Why oblique derivative boundary value problem ?

- Unusual boundary condition.
- It appears in the modeling of some mechanical problems, but perhaps not directly under the stated form; see Mehats (Convergence of a numerical scheme for a nonlinear oblique derivative boundary value problem. M2AN, 2002) and references therein.
- In Mehats, a finite difference scheme is proposed and analyzed.

First contribution (suite): Finite volume for oblique boundary value problems.

Description of this work. We add the known condition in order to get the well-posedness $\int_\Omega u(\boldsymbol{x})d\boldsymbol{x} = 0$:

- We provide a finite volume scheme under an admissible mesh for the problem under consideration.
- We prove the convergence under weak regularity towards exact of a weak formulation.
- We provide a convergence rate.
- We also addressed the case when α is a smooth function or is constant on each segment of the boundary $\partial\Omega$.

Second contribution: Finite volume for an ohmic losses problem.

Subject of this work:

Discretization of the coupled heat and electrical diffusion problems by the finite element and the finite volume methods. With R. Herbin. IMAJNA, 2008.

Model problem

Find the solution (u, ϕ) of the nonlinear coupled elliptic system, which models the thermal and electrical diffusion in a material subject to ohmic losses:

$$-\nabla \cdot (\kappa(\boldsymbol{x}, u(\boldsymbol{x}))\nabla \phi(\boldsymbol{x})) = f(\boldsymbol{x}, u(\boldsymbol{x})), \;\; \boldsymbol{x} \in \Omega, \tag{11}$$

$$-\nabla \cdot (\lambda(\boldsymbol{x}, u(\boldsymbol{x}))\nabla u(\boldsymbol{x})) = \kappa(\boldsymbol{x}, u(\boldsymbol{x}))|\nabla \phi|^2(\boldsymbol{x}), \;\; \boldsymbol{x} \in \Omega, \tag{12}$$

with $\phi(\boldsymbol{x}) = u(\boldsymbol{x}) = 0$, for all $\boldsymbol{x} \in \partial\Omega$.

Second contribution (suite): Finite volume for an ohmic losses problem.

Why this system?

- Is a nonlinear system of elliptic equations, which arises when modelling the heat diffusion problem coupled with the electrical diffusion problem.
- Some theoretical results concerning the existence are already known in Gallouët and R. Herbin (Existence of a solution to a coupled elliptic system, Appl. Math. Lett, 1994).
- Some numerical results related to the stated problem are already exist in Ferguson, J. M. Fiard and R. Herbin (A mathematical model of solid oxide fuel cells, Journal of Power Sources, 1996.

Second contribution (suite): Finite volume for an ohmic losses problem.

Description of this work:

- A finite element scheme and a finite volume scheme (using admissible meshes) are considered for the discretization of the system.
- In both cases, the convergence of approximate solutions obtained, up to a subsequence, to a solution of the coupled elliptic system is shown.

Third and fourth contributions: Finite volume for parabolic equation.

Subject of these two works:

- Some abstract error estimates of a finite volume scheme for a nonstationary heat equation on general nonconforming multidimensional spatial meshes. With J. Fuhrmann. Appl. Math., Praha, 2013.

- An analysis of a second order time accurate scheme for a finite volume method for parabolic equations on general nonconforming multidimensional spatial meshes. Appl. Math. Comput, 2013.

Model problem: nonstationary Heat equation:

$$u_t(\boldsymbol{x},t) - \Delta u(\boldsymbol{x},t) = f(\boldsymbol{x},t), \ \boldsymbol{x} \in \Omega, t \in (0,T).$$

Third and fourth contributions (suite): Finite volume for parabolic equations.

Why this study and a description of the results.

- Some convergence results, using SUSHI scheme, are already known for the time-independent case in Eymard, Gallouët, and Herbin **(IMAJNA, 2010)**.
- The model we studied is the heat equation which describes the distribution of heat (or variation in temperature) in a given region over time.
- We provide schemes of order one/two in time using SUSHI scheme.
- Error estimates are performed $H^1(L^2)$, $W^{1,\infty}(L^2)$, and $L^\infty(L^2)$ norms.

Fifth contribution: Finite volume for second order hyperbolic equation.

Subject of this work:

Convergence analysis of some high–order time accurate schemes for a finite volume method for second order hyperbolic equations on general nonconforming multidimensional spatial meshes. Numer. Methods Partial Differ. Eq, 2013.

Model problem: Wave equation:

$$u_{tt}(\boldsymbol{x},t) - \Delta u(\boldsymbol{x},t) = f(\boldsymbol{x},t),\ \boldsymbol{x} \in \Omega, t \in (0,T).$$

Fifth contribution (suite): Finite volume for second order hyperbolic equation.

Why this study and a description of the results.

- Some convergence results, using SUSHI scheme, are already known for the time-independent case in Eymard, Gallouët, and Herbin **(IMAJNA, 2010)**.
- The model we studied is the Wave equation which describes waves–as they occur in physics–such as sound waves, light waves and water waves. It arises in fields like acoustics, electromagnetics, and fluid dynamics (from Wikipedia).
- We provide schemes of order two or more in time using SUSHI scheme.
- Error estimates are performed $H^1(L^2)$, $W^{1,\infty}(L^2)$, and $L^\infty(L^2)$ norms.

Introduction

Finite element (FE) methods are popular and they are used to approximate partial differential equations. One the main properties of FE is that they are based on the use of Weak formulation of the problem to be solved. Two main spaces are used: the space (set) in which lies of the exact solution and the space of test functions.

We were interested with the convergence order of finite element approximate solutions for time dependent problems using either Crank-Nicolson or Newmark methods as discretization in time. More precise, we addressed:

- Convergence order of Conforming FE for Parabolic equations when the time discretization is performed using Crank-Nicolson method.
- Convergence order of Conforming FE for Second order Hyperbolic equations when the time discretization is performed using Newmark method.

Contributions related to second axis"Convergence order in Finite Element methods"

Subject of these contributions:

- A new error estimate for a Crank–Nicolson finite element scheme for parabolic equations. With Fuhrmann. Mathematica Bohemica, 2014.
- Some new error estimates for finite element methods for the acoustic wave equation using the Newmark method. With Fuhrmann. Mathematica Bohemica, 2014.

Model problems: Heat and Wave equations:

$$u_t(\boldsymbol{x},t) - \Delta u(\boldsymbol{x},t) = f(\boldsymbol{x},t),\ \boldsymbol{x} \in \Omega, t \in (0,T)$$

and

$$u_{tt}(\boldsymbol{x},t) - \Delta u(\boldsymbol{x},t) = f(\boldsymbol{x},t),\ \boldsymbol{x} \in \Omega, t \in (0,T)$$

Description of the two works related to second axis “Convergence order in Finite Element methods”

Description of these contributions:

- We consider conforming finite element methods as discretization in space.
- We use Crank-Nicolson (resp. Newmark) method as discretization in time for Heat (Wave) equation.
- We prove error estimates in $H^1(L^2)$, $W^{1,\infty}(L^2)$.

Introduction

Usually, when we would like to obtain high convergence numerical approximations, we have either to use higher order schemes (which includes several mesh points) or to increase the degree of finite element spaces. This leads in general to systems to be solved with many non zero entries. Consequently, the computational cost will be increased.

The aim then is:

- **To construct high order numerical approximations (We focus on the finite volume methods).**
- **The matrices involved to compute these high order approximations are simple.**

Description of the approach to get high order numerical approximations

First step: we compute initial (basic) solution u_h

Using a low-order scheme, we obtain the following linear system to be solved:

$$\mathcal{A}^{\mathcal{T}} u_h = f_h. \tag{13}$$

Second step: we compute a new high order approximation u_h^1

We correct the right hand side of (13) in order to get high order approximation by resolving the system (13) with the same matrix and changing only the right hand side, i.e.:

$$\mathcal{A}^{\mathcal{T}} u_h^1 = f_h + \delta f_h. \tag{14}$$

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Advantages of the approach

First advantage

Usually when using low order schemes, the matrices $\mathcal{A}^{\mathcal{T}}$ used to compute the numerical solution u_h are sparse, e.g. linear finite element method and central three point scheme.

Second advantage

The matrix $\mathcal{A}^{\mathcal{T}}$ used to compute the new high order approximation u_h^1 is the same one used to compute the initial solution u_h. This means that the computational costs of u^h and u_1^h are comparable.

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Contributions related to high order finite volume approximations

- A theoretical analysis of a new second order finite volume approximation based on a low–order scheme using general admissible spatial meshes for the one dimensional wave equation. JMMA (Journal in Mathematical Analysis and Applications), 2015.
- A full analysis of a new second order finite volume approximation on a low order scheme using general admissible spatial meshes for the unsteady one dimensional heat equation. JMMA (Journal in Mathematical Analysis and Applications), 2014.

Description of the contributions related to high order finite volume approximations

- We provide an approach allowing to construct high order finite volume approximations for Heat and Wave equations in 1D using some simple tridiagonal matrices.
- The convergence analysis of these high order finite volume approximations is performed in several discrete norms.

Introduction on COMSOL

COMSOL Multiphysics (formerly FEMLAB) is a commercial Software based on the use of Finite Element methods. COMSOL Multiphysics simulation software environment facilitates all steps in the modeling process: starting by defining the geometry (physical domain), meshing, specifying the physics, solving, and then visualizing the results.

One of the advantages of the COMSOL Multiphysics is that there is a script (the comsol script) which allow us to do some programs. COMSOL Script is a numerical computing and programming environment mainly used for interacting with finite element multiphysics models created by COMSOL Multiphysics. It uses a syntax similar to that of MATLAB.

Contributions related to COMSOL Multiphysics

- On the convergence order of the COMSOL solutions in Sobolev norms. With Holzbecher. CD Proceedings of the COMSOL Conference of Budapest, November 2008.
- On the convergence order of the COMSOL solutions. With Holzbecher. CD Proceedings of the COMSOL Conference of Grenoble, October 2007.

A description of the contributions related to COMSOL Multiphysics

Using the COMSOL Script, we checked that the convergence order of the COMSOL solution is the same one as in the theoretical framework of Finite Element methods.

Works under preparation and perspectives

Numerical schemes for time dependent Fractional PDEs.

High order finite volume approximations in 2D and 3D.

As perspectives

COMSOL Multiphysics to check some known results in FE for complex equations, e.g. Navier Stokes equation.

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