Abstract for the HDR thesis presented by

Dr. A. Bradji:

Contribution à l' Etude de Convergence de Schémas de Discrétisation d' Equations aux Dérivées Partielles

The thesis includes three items:

- Convergence order of Finite Element methods and Finite Volume methods.
- High convergence order of numerical methods using low order schemes.
- Convergence order of the COMSOL solutions.

Let us now describe in brief the above items.

1 Convergence order of Finite Element methods and Finite Volume methods

As we know that the convergence order of numerical methods measures the *speed* of the schemes under consideration. Consequently, the convergence order is important from *mathematical point of view* and also in *practice* in which we aim to get an approximation very close to the exact solution. One of the our goals is to obtain some results of convergence order of numerical schemes, in finite element and finite volume methods, for some known models in Physics.

2 High convergence order of numerical methods using low order schemes

Finite volume methods have been widely used for the numerical simulation of various types (elliptic, parabolic or hyperbolic equations) of conservation laws. Some of the important features of finite volume method are similar to those of finite element method: it may be used on arbitrary geometries, using structured or unstructured meshes, and leads to robust schemes.
Let us describe our *approach* to get high order approximations in the context of Finite volume methods. Let

$$\mathcal{L}u = f, \tag{1}$$

be an elliptic equation posed on a sufficiently smooth domain Ω of $\mathbb{R}^d$, $d = 1$, 2 or 3 .
We consider an "admissible mesh" $\mathcal{T}$. Thus the finite volume approximation of the equation (1) leads to a system to be solved:

$$\mathcal{L}^{\mathcal{T}} u^{\mathcal{T}} = f^{\mathcal{T}}, \tag{2}$$

where $\mathcal{L}^{\mathcal{T}}$ is a "good" matrix (in terms of computational cost) and $u^{\mathcal{T}}$ is the finite volume approximate solution.
In general, the convergence order of the finite volume solution $u^{\mathcal{T}}$ is $O\left(\text{size}(\mathcal{T})\right)$.
To get higher convergence order, we have to use, in general, high order schemes. This leads to complex systems to be solved.
Nevertheless, we can use the "basic" finite volume solution $u^{\mathcal{T}}$ to get new finite volume approximations of higher convergence orders, successively. These new approximations can be computed using the same matrix $\mathcal{L}^{\mathcal{T}}$ and changing only the second members of the systems we resolve. Thus the computational costs of these approximations are "comparable" to that of the basic finite volume solution $u^{\mathcal{T}}$.

Some of my research interests are concerned with the techniques which allow us to obtain high order approximations (in Finite Volume methods and Finite Element methods) on arbitrary meshes without make appeal to the standard methods which use high order schemes (in Finite Volume methods) or finite element spaces with high degree polynomials.

3 Convergence order of the COMSOL solutions

COMSOL Multiphysics (formerly FEMLAB) is a commercial Software based on the use of Finite Element methods, see http://en.wikipedia.org/wiki/COMSOL_Multiphysics. COMSOL Multiphysics simulation software environment facilitates all steps in the modeling process: starting by defining the geometry (physical domain), meshing, specifying the physics, solving, and then visualizing the results. One of the advantages of the COMSOL Multiphysics is that there is a script (the comsol script) which allow us to do some programs. COMSOL Script is a numerical computing and programming environment mainly used for interacting with finite element multiphysics models created by COMSOL Multiphysics. It uses a syntax similar to that of MATLAB.

Using the COMSOL Script, we checked that the convergence order of the COMSOL solution is the same one as in the theoretical framework of Finite Element methods.

4 Contributions

4.1 Submitted for publication

[B-S1] *Convergence analysis of some new first order and second order time accurate finite volume gradient schemes for semilinear parabolic equations in any space dimension.*

[B-S2] *Convergence analysis of a gradient scheme for a semilinear second order hyperbolic equation.*

[B-S3] *Error estimates of finite volume gradient schemes for elliptic and parabolic equations with linear oblique derivative boundary conditions.*

4.2 Articles in Journals

[B1] *A theoretical analysis for a new finite volume scheme for a linear Schrödinger evolution equation on general nonconforming spatial meshes.* Numerical Functional Analysis and Optimization, 36/5, 590–623, 2015.

[B2] *A theoretical analysis of a new second order finite volume approximation based on a low–order scheme using general admissible spatial meshes for the one dimensional wave equation.* JMAA (Journal in Mathematical Analysis and Applications), 422/1, 109–147, 2015..

[B3] *A full analysis of a new second order finite volume approximation on a low order scheme using general admissible spatial meshes for the unsteady one dimensional heat equation.* JMAA (Journal in Mathematical Analysis and Applications), 416/1, 258–288, 2014.

[B4] *Some new error estimates for finite element methods for the acoustic wave equation using the Newmark method. With Jürgen Fuhrmann.* Mathematica Bohemica, 139/2, 125–136, 2014

[B5] *A new error estimate for a Crank–Nicolson finite element scheme for parabolic equations.* With Jürgen Fuhrmann. Mathematica Bohemica, 139/2, 113–124, 2014.

[B6] *An analysis of a second order time accurate scheme for a finite volume method for parabolic equations on general nonconforming multidimensional spatial meshes.* Applied Mathematics and Computation, 219/11, 6354–6371, 2013.

[B7] *Convergence analysis of some high–order time accurate schemes for a finite volume method for second order hyperbolic equations on general nonconforming multidimensional spatial meshes.* Numerical Methods for Partial Differential Equations, 29/4, 1278–1321, 2013.

[B8] *A theoretical analysis of a new finite volume scheme for second order hyperbolic equations on general nonconforming multidimensional spatial meshes.* Numerical Methods for Partial Differential Equations, 29/1, 1–39, 2013.

[B9] *Some abstract error estimates of a finite volume scheme for a nonstationary heat equation on general nonconforming multidimensional spatial meshes.* With J. Fuhrmann. Applications of Mathematics, Praha, 58/1, 1–38, 2013.

[B10] *Error estimates of the discretization of linear parabolic equations on general nonconforming spatial grids.* With J. Fuhrmann. Comptes rendus - Mathématique 348/19-20 , 1119–1122, 2010.

[B11] *Some simples error estimates for finite volume approximation of parabolic equations.* Comptes Rendus de l'Académie de Sciences, Paris, 346/9-10 pp. 571-574, 2008.

[B12] *Discretization of the coupled heat and electrical diffusion problems by the finite element and the finite volume methods.* With R. Herbin. IMA Journal of Numerical Analysis, **28** (3), 469–495, 2008.

[B13] *Optimal defect corrections on composite nonmatching finite element meshes.* With A.-S. Chibi. IMA Journal of Numerical Analysis, **27** (4), 765– 780, 2007

[B14] *Error Estimate for Finite Volume Approximate Solutions of Some Oblique Derivative Boundary Problems.* With T. Gallouët. International Journal on Finite Volumes. **3** (2), 35 pages (electronic), 2006

[B15] *Impropved Convergence Order for Finite Volume Solutions. Part I: 1D Problems.* With B. Atfeh. Arab Journal of Mathematical Sciences. **11** (1), 1–30, 2005.

[B16] *Impropved Convergence Order for Finite Volume Solutions. Part II: 2D Problems.* With B. Atfeh. Arab Journal of Mathematical Sciences. **11** (2), 1–53, 2005.

4.3 Articles in Peer Reviewed Proceedings

[B-P1] *Note on a new piecewise linear finite element approximation of order four for one dimensional second order elliptic problems on general meshes.* Proceedings of MAMERN VI 2015, 175–185 (2015), B. Amaziane et al. (Eds), ISBN 978-84-338-5783-5

[B-P2] *Some discrete a priori estimates for a finite volume scheme appearing in the discretization of a time dependent Joule heating system.* Proceedings of MAMERN VI 2015, 187–197 (2015), B. Amaziane et al. (Eds), ISBN 978-84-338-5783-5

[B-P3] *A convergence order for a finite volume scheme for a semilinear parabolic equation.* Proceedings of MAMERN VI 2015, 199–2010 (2015), B. Amaziane et al. (Eds), ISBN 978-84-338-5783-5

[B-P4] *Note on the convergence of a finite volume scheme using a general nonconforming mesh for an oblique derivative boundary value problem.* Springer Proceedings in Mathematics and Stochastics, V. 77, 2014, 149–157: Finite Volumes for Complex Applications VII, Methods and Theoretical Aspects (Fuhrmann et al. Eds.).

[B-P5] *A new finite volume scheme for a linear Schrödinger evolution equation.* Springer Proceedings in Mathematics and Stochastics, V. 77, 2014, 127–135: Finite Volumes for Complex Applications VII, Methods and Theoretical Aspects (Fuhrmann et al. Eds.).

[B-P6] *A note on a new second order approximation based on a low-order finite volume scheme for the wave equation in one space dimension.* Springer Proceedings in Mathematics and Stochastics, V. 77, 2014, 137–147: Finite Volumes for Complex Applications VII, Methods and Theoretical Aspects (Fuhrmann et al. Eds.).

[B-P7] *Some second order time accurate for a finite volume method for the wave equation using a spatial multidimensional generic mesh.* Handlovicova et al. (ed.), Algoritmy 2012. Proceedings of contributed papers and posters. Bratislava: Slovak University of Technology, Faculty of Civil Engineering. 342–352 (2012).

[B-P8] *Some abstract error estimates of a finite volume scheme for the wave equation on general nonconforming multidimensional spatial meshes.* Proceedings of International Symposium of Finite Volume for Complex Applications Edited by J. Fort et al., Springer Proceedings in Mathematics 4, 175–183, 2011.

[B-P9] *Some Error Estimates for the Discretization of Parabolic Equations on General Multidimensional Nonconforming Spatial Meshes.* With J. Fuhrmann. I. Domov et al. (Eds), NMA 2010, LNCS 6046, 369–376, 2011. Springer–Verlag, Berlin Heidelberg 2011.

[B-P10] *Towards an approach to improve convergence order in finite volume and finite element methods.* Proceedings of ICNAAM "International Conference in Numerical Analysis and Applied Mathematics", Edited by T. E. Simos, G. Psihoyios, and Ch. Tsitouras, 1162–1165, 2009

[B-P11] *Some error estimates in finite volume methods for parabolic equations.* With J. Fuhrmann. Finite Volumes for Complex Applications V, Proceedings of the 5th International Symposium on Finite Volume for Complex Applications/ edited by R. Eymard and J.-M. Hérard, Wiley, 233–240, 2008.

[B-P12] *On the discretization of Ohmic losses.* With R. Herbin. Proceedings of Tamtam, 2007,Tipaza, Algeria, 217–222. AMNEDP-USTHB, 2007.

[B-P13] *On the discretization of the coupled heat and electrical diffusion problems.* With R. Herbin. Numerical Methods and Applications. 6 th International Conference, NMA 2006, Borovets, Bulgaria, Aug. 20–24, 2006. Lecture Notes in Computer Science 4310 Springer 2007, pp. 1–15.

[B-P14] *Finite volume approximation for an oblique derivative boundary problem.* with T. Gallouët. Finite Volumes for Complex Applications IV, Proceeding of the 4th International Symposium on Finite Volume for Complex Applications/edited by F. Benkhaldoun, D. Ouazar, and S. Raghay, Hermes-Penton, pp. 143–152, 2005.

[B-P15] *Improved convergence order of finite solutions and application in finite elements methods.* Proceedings of ICNAAM: International Conference in Numerical Analysis and Applied Mathematics, Simos, G. Psihoyios and C. Tsitouras (eds), Wiley -VCH, pp. 94-98, 2005.

4.4 Contributions using COMSOL Multiphysics

[B-COM1] *On the convergence order of the COMSOL solutions in Sobolev norms.* With Holzbecher. CD Proceedings of the COMSOL Conference of Budapest, November 2008.

[B-COM2] *On the convergence order of the COMSOL solutions.* With Holzbecher. CD Proceedings of the COMSOL Conference of Grenoble, October 2007.