A brief Report on the article [KU 10]"Pointwise error estimates for a stabilized Galerkin method: non-selfadjoint problems"

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Abstract: The authors provides highly localized pointwise error estimates for a stabilized Galerkin method proposed in some years ago. Stabilization terms are added to the standard method to guarantee the existence and uniqueness of the approximate solution regardless of meshsize. The stabilized Galerkin method leads to a minimization problem requiring the solution of a symmetric and positive definite system. Compared to the standard Galerkin method, the stabilized Galerkin method considered in this paper provides a better numerical performance on a coarse mesh and about the same performance on a fine mesh. Recently, highly localized pointwise error estimates are obtained for selfadjoint elliptic partial differential equations for the stabilized Galerkin method. However, the argument cannot be generalized to non-selfadjoint problems. The authors treats then the case of non-selfadjoint problems. He first proves that the stabilized Galerkin solution is superclose (by the factor of h) to the Ritz projection of the true solutions. Then, he proceeds to obtain the pointwise estimate using the discrete Green's function. Application to error expansion inequalities and a posteriori error estimators are discussed.

Key words and phrases: Stabilized Galerkin method; Pointwise error estimates; non-selfadjoint problems; discrete Green's function; error expansion inequalities; posteriori error estimators

Subject classification:65N30; 65N15; 65N12; 35J25

Remark 1 The article [KU 10] under consideration is well written and deserves more reading. Because of time, I could only summarized the following information and my hope I get some free time to come back to this useful article.

1 Basic information, an overview, and some remarks

The authors provides highly localized pointwise error estimates for a stabilized Galerkin method proposed in [BRA 98] and [KU 07]. Stabilization terms are added to the standard Galerkin method

to guarantee the existence and uniqueness of the approximate solution regardless of meshsize. It is well known, see [SCH 98], that the meshsize needs to be sufficiently small in order to guarantee the existence and uniqueness of the standard Galerkin method for non-coercive problems. The stabilized Galerkin method leads to a minimization problem requiring the solution of a symmetric and positive definite system.

References

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