A brief Report on the article "High–order, finite-volume methods in mapped coordinates "

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Abstract: The aim of this article is to provide a new approach to get higher order finite volume approximatios. Among the main ideas of the article is to replace the integrands which appears after integration of the divergence flux on the control volumes by Taylor expansions about the center of faces and then replacing the derivatives by finite-difference approximations of a suitable order that are smooth functions of their inputs. This approach has been used for elliptic and hyperbolic equation when the physical domain in is mapped into a cube. Numerical examples are presented to justify the theoretical results.

Key words and phrases: Finite volume method; Higher order method; Mapped grids; Hyperbolic and elliptic equations

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1 basic idea

Let us consider a finite volume mesh based on rectangular meshs as in [EYM 00] and we consider the approximation of the following divergence expression in two dimension:

$$\nabla \cdot F \tag{1}$$

Let us consider the control volume $K_{ij} =]x_{i-\frac{1}{2}}, x_{i+\frac{1}{2}}[\times]y_{j-\frac{1}{2}}, y_{j+\frac{1}{2}}[$. Integrating the first component of the expression [1] on K_{ij}, we get

$$\int_{y_{j-\frac{1}{2}}}^{y_{j+\frac{1}{2}}} (u(x_{i+\frac{1}{2}}, y) - u(x_{i-\frac{1}{2}}, y))dy \tag{2}$$

Let us consider the following Taylor's expansion

$$\begin{aligned}\int_{y_{j-\frac{1}{2}}}^{y_{j+\frac{1}{2}}} (u(x_{i+\frac{1}{2}}, y) - u(x_{i-\frac{1}{2}}, y))dy &= k_j(u(x_{i+\frac{1}{2}}, y_j) - u(x_{i-\frac{1}{2}}, y_j)) + \mathrm{Cte}(u_{yy}(x_{i+\frac{1}{2}}, y_j) - u_{yy}(x_{i-\frac{1}{2}}, y_j)) \\ &+ 0(h^4),\end{aligned} \tag{3}$$

where $y_j = (y_{j-\frac{1}{2}} + y_{j+\frac{1}{2}})/2$.

The idea is that If we replace the derivatives by finite-difference approximations of a suitable order that are smooth functions of their inputs, the resulting approximation of the average of the flux divergence over a cell is of higher order

2 headlines of the article

1. assume that we have a problem posed on physical domain $\Omega \subset \mathbb{R}^d$ and we assume that there is a smooth mapping from D to the cube $(0,1)^d$
2. computations done on $(0,1)^d$ can be mapped to Ω; in particular how to get higher order schemes on $(0,1)^d$.
3. application to elliptic problems
4. application to hyperbolic problems

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