A BRIEF REPORT ON THE ARTICLE "NEW PROPERTIES OF 9-POINT FINITE DIFFERENCE SOLUTION OF THE LAPLACE EQUATION -A. A. DOSIYEV"

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Abstract. The author considers a finite difference scheme of 9 points on square grids to approximate the Laplace equation on rectangular region with Dirichlet boundary conditions. The case when the boundary functions are in $C^{5,1}$ is discussed. The necessary differential properties of the exact solution are also given. Two properties for the finite difference scheme considered by the author are derived. The first property is that the order is $h^6(1+\ln|h|)$, in the maximum norm. The second property is that the stated order can not be obtained when for instance the boundary functions are only in the Hölder spaces $C^{5,\lambda}$, where $0<\lambda<1$. This is proved thanks to a suitable estimation below.

Last update: Wednesday 28th March, 2012. My hope I come back to learn from this article and from other articles written by the author Dosiyev.

1. some basic knowledge

1. the present work is a continutation of his previous work [1, 2, 3, 4, 5]
2. the reference [7] seems a very nice and useful reference for many works related to the numerical methods for pdes
3. useful references quoted by the author
4. the present work is done on rectangular region. may be it useful to move to a polygonal region in two dimensions. this may be requires some additional techniques.
5. the present work deals with finite difference methods. what about finite volume or finite element methods? I think this could be an interesting task to study !
6. useful literature quoted by the author in the introduction.

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