A BRIEF REPORT ON THE ARTICLE "ON THE CONSTRUCTION OF WELL CONDITIONED HIERARCHICAL BASES FOR TETRAHEDRAL H(CURL) CONFORMING NEDELEC ELEMENTS-BY JIANGUO XIN ET AL"

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Abstract. A new set of hierarchical basis for tetrahedral H(curl)-conforming elements has been proposed with the goal of improving the conditioning of the mass and quasi-stiffness matrices. The basis functions are given analytically. The construction of the new basis is motivated by the study of orthogonal polynomials of several variables, and based upon the work of some authors. The idea is to make each sub-set of shape functions, grouped and associated with a topological entity on the 3-simplex, have maximum orthonormality over the reference element. This is achieved by appropriately exploiting classic orthogonal polynomials, viz., Legendre and Jacobi polynomials over simplicial elements. The result of such a construction is that the basis functions are partially orthonormal over the 3-simplex but not completely. One is tempted to use the standard Gram-Schmidt orthogonalization procedure to make the entire basis functions orthonormal. However, such an effort will destroy the unique features of each category to which a particular set of basis functions belong. In this sense, explicit construction of a hierarchical and complete orthonormal basis for tetrahedral H(curl)-conforming elements does not exist.

The sparsity pattern of the mass and quasi-stiffness matrices has been studied numerically, and the opposite trend of percentage of nonzero entries in both matrices has been identified. Compared with a previous work for Ainsworth and Coyle basis and in general, both the mass and quasi-stiffness matrices are relatively more sparse. The numerical experiment has shown that the conditioning of the mass matrix is relatively more pronounced, i.e., one order higher than that with the quasi-stiffness matrix. For both the mass and quasi-stiffness matrices and on the logarithmic scale, the condition number grows linearly vs. the order of approximation up to order three. It does not make too much sense if a rather high-order basis, e.g., beyond order six is applied due to the quick growth of the condition numbers of the mass and quasi-stiffness matrices.

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