A brief Report on the article [BOY 10] "Nonoverlapping Schwarz algorithm for solving two-dimensional m-DDFV schemes."

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Last update: Tuesday 28th December, 2010; not finished yet and my hope I come back again to this article

Abstract: This article deals with a m-DDFV (modified discrete duality finite volume) scheme for anisotropic elliptic problems with mixed Dirichlet/Fourier boundary conditions. As a result, the authors provide a nonoverlapping Schwarz algorithm associated with a subdomain decomposition of the problem domain for solving m-DDFV scheme on the whole domain. The convergence of the Schwarz algorithm is proved to converge to the solution of the m-DDFV scheme on the whole domain. The properties of the algorithm were illustrated by numerical results on anisotropic elliptic equations.

Key words and phrases: finite volume methods; Nonoverlapping Schwarz algorithm; discrete duality finite volume (DDFV) schemes; mixed Dirichlet/Fourier boundary conditions

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Some remarks

1. idea on m-DDFV: m-DDFV ("m" is for modified) is a discrete duality finite volume introduced to take into account the possible discontinuities in the coefficients of the elliptic equation under study. It is first introduced in [BOY 08].

   The DDFV methos has been developed to approximate anisotropic diffusion problems on general meshes. It was first introduced and studied in [HER 03, DOM 05] to approximate the Laplace equation with Dirichlet boundary conditions or homogeneous Neumann boundary conditions on a general class of meshes.

2. equation solved: the following problem is considred in [BOY 10]:

$$-\nabla\cdot(\Lambda(x)\nabla u(x)) = f(x),\ x\in\Omega, \qquad [1]$$

   with the following boundary conditions

(a) Dirichlet boundary conditions

$$u(x) = h(x),\ x \in \partial\, x \in \Omega \setminus \Gamma, \qquad [2]$$

(b) Fourier boundary conditions

$$-\left(\Lambda(x) \cdot \nabla\, u(x)\right) = \lambda\, u(x) - g(x),\ x \in \Gamma, \qquad [3]$$

where Ω is an open bounded polygonal domain of $\mathbb{R}^2$. The measurable matrix–valued map $\Lambda : \Omega \to \mathcal{M}_{2,2}$ is assumed to fulfil the following assumptions: there exists $C_A > 0$ such that

$$(\,\Lambda(x)\xi, \xi) \geq \frac{1}{C_A}|\,\xi|^2 \ \text{ and } |\,\Lambda(x)\xi| \leq C_A|\,\xi|,\ \ \forall\, \xi \in \mathbb{R}^2,\ \forall\ a.e.\ x \in \Omega, \qquad [4]$$

$f \in H^{-1}(\Omega)$, and $g, h \in H^{\frac{1}{2}}(\Omega)$. .

References

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