A brief Report on the article [HOV 10]"On the stability of fully adaptive multiscale schemes for conservation laws using approximate flux and source reconstruction strategies "

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Last update: Friday 31st December, 2010; It seems that I should come back again to read more this article

Abstract: The aim of this article is to propose an approximate flux and source reconstruction strategy. The basic idea is to compute, for each cell in the adaptive grid, a reconstruction polynomial by which the authors provide the data for the computation of the local fluxes. Moreover, the local sources are determined by a quadrature rule applied to the composition of the source function and the reconstruction polynomial. It is proved that this strategy does not spoil the computational complexity of the adaptive scheme even in higher dimensions. Furthermore, it is verified analytically that in this way the accuracy of the reference scheme can be maintained, i.e., the perturbation error introduced by the adaptive procedure is controlled by the threshold parameter in such a way that is the same order as the discretization error of the reference scheme. In particular, by evolving the partial differential equation on adaptive grid using the approximate reconstruction strategy, the authors introduced an additional error in comparison with the evolution with exact reconstruction. This error is proportional to the threshold value. The theoretical results are confirmed by numerical parameter studies.

Key words and phrases: conservation laws; finite volume schemes; fully adaptive multiscale; numerical flux; biorthogonal wavelets

Subject Classification : 65N08; 35L65; 65T60; 65M50

1 What i have learned from the article

1. some overwiew: it seems for that the article is full by many concepts: from finite volume, passing by adaptive mesh, to the use of wavelets; my hope I come back again to learn more from this article.

2. some beautiful knoledge on finite volume: Nowdays finite volume methods are frequently used

for the discretization of conservation laws as they arise, for instance, in CFD (Computational Fluid Dynamics).

3. approach used: the approach used here is Harten's original approach on the [HAR 95] .

4. equation solved: the following equation is considered in [HOV 10]:

$$u_t(t,x) + f(u(t,x))_x = s(u(t,x)),\ t > 0,\ x \in \mathbb{R}, \qquad [1]$$

subject to the initial condition

$$u(0,x) = u_0(x),\ x \in \mathbb{R}. \qquad [2]$$

References

[ABG 97] R. Abgrall: Multiresolution analysis on unstructured meshes: applications to CFD. *Experimentation, Modeling and Computation in Flow, Turbulence and Combustion (Chetverushkin et al. eds.)* Computational Methods in Applied Sciences, Vol. **2**. Chichester, UK: Wiley, 147–156, 1997.

[COH1 00] A. Cohen: Wavelet Methods in Numerical Analysis, *Ciarlet, P. G. (ed.) et al., Handbook of numerical analysis. Vol. 7: Solution of equations in* $\mathbb{R}^n$ *(Part 3). Techniques of scientific computing (Part 3). Amsterdam: North-Holland/ Elsevier.* 417–711, 2000.

[COH 92] A. Cohen, I. Daubechies, and J. Feauveau: Bi–orthogonal bases of compacity supported wavelet , *Commun. Pure Appl. Math.*, **45**, 485–560, 1992.

[COH2 00] A. Cohen, N. Dyn, S. M. Kaber, and M. Postel: Multiresolution schemes of triangles for scalar conservation laws. *J. Comput. Phys.*, **161** (1), 264–286, 2000.

[COH 03] A. Cohen, N. Dyn, S. M. Kaber, M. Postel, and S. Müler: Fully adaptive multiresolution finite volume schemes for conservation laws. *Math. Comput.*, **72** (241), 183–225, 2003.

[HAR 95] A. Harten: Multiresolution algorithms for numerical solution of hyperbolic conservation laws. *Commun. Pure Appl. Math.*, **48**, 1305–1342 (1995).

[HOV 10] N. Hovhannisyan, and S. Müller: On the stability of fully adaptive multiscale schemes for conservation laws using approximate flux and source reconstruction strategies. *IMA J. Numer. Anal.*, **30**, No. 4, 1256–1295 (2010).