A brief Report on the article [BRU 10]"Computational solution of blow-up problems for semilinear parabolic pdes on unbounded domains "

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Abstract: A numerical treatement of a model of semilinear parabolic equations on unbounded domain where the solutions blow up in finite time is presented. The numerical approximation requires selecting an appropriate bounded computational domain. The authors present suitable local absorbing boundary conditions (LABCs) in order to derive the bounded computational domain. A unified approach to derive such LABCs for one dimensional and 2D spatial domains is presented. Numerical results to illustrate theoretical results are provided.

Key words and phrases: blow up problems; semilinear parabolic equation; unbounded domains; Laplace transformation; nonlinear absorbing boundary conditions; artificial boundary conditions; local absorbing boundary conditions. A unified approach

Subject Classification : 65M06; 65L10; 65Q53; 65Q51

1 What i have learned from the article

1. some overwiew: the article considers the following problem:

$$u_t(x,t) - \Delta\, u(x,t) = u^p(x,t),\ t > 0,\ x \in \mathbb{R}^d, \qquad [1]$$

   subject of initial condition $u(x,0) = u_0(x)$ where $d = 1$ or 2 and $p > 1$.

2. some review: while the theory of finite time blow up for problem [1] is well understood (cf. [SAM 95], the numerical analysis of blow up on unbounded spatial domains has so far received little attention.

3. some literature: first contributions in the subject of numerical methods for blow up problems are those of [NAK 75, NAK 77] in which the spatial domain is bounded. [NAK 75, NAK 77] used finite difference and finite element methods.

4. difficulties: the essential difficulties to approximate [1] consist of the following three part:

   (a) the nonlinearity

(b) the unboundedness of the spatial domain

(c) the multidimentiality of the spatial domain

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