A brief Report on the article [YUA 11]"Sharp a posteriori error estimate for elliptic equation with singular data"

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Last update: Monday 9th May, 2011; not finished yet and my hope to come back to this article to learn more.

Abstract: The authors consider the finite element approximation of the Laplace equation, in two dimensions, with right–hand side in L^p space with $1 < p \leq 2$. The authors derived two a posteriori error estimators. The first estimator is a residual based a posteriori error estimator, including the both the bulk residual and the edge residual. The second one consists of only the edge residual in terms of the solution of the local residual problem. Both estimators are proved to be the global upper and local lower bounds on the error in $W^{1,p}$-seminorm. It is proved numerically that the estimators lead to optimal convergence orders.

Key words and phrases: second order elliptic equation; two dimensions; finite element method, sharp a posteriori error estimate; singular data

Subject Classification : 65N30; 65N15

1 Some questions....

1. what about the case when right hand side is only in $\mathbb{L}^1$

2 An overview

Let us consider the Laplace equation with right hand side denoted by f. Consider a linear finite element to approximate the problem. It is well known that when the problem domain is smooth and f is in $\mathbb{L}^2$, then the exact solution is in the Sobolev space H^2. So, an optimal error estimate of order h in H^1–norm, and h^2 in $\mathbb{L}^2$–error. When the data is not $\mathbb{L}^2$, at least two difficulties arise in order to determine the convergence order:

1. A weak formulation for problem
2. The regularity of the solution

The previous items are treated for instance, in my knowledge, in [BRA 08, CAS 07, CLA 96, GAL 05, GAL 04, SCO 73].
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