# Some notes on the article [ZHU 11]"Energy norm a posteriori error estimation for $hp$-adaptive discontinuous Galerkin methods for elliptic problems in three dimensions"

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**Math. Models Methods Appl. Sci. 21, No. 2, 267–306 (2011)**

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Provisional home page: http://www.cmi.univ-mrs.fr/~bradji

Last update: Tuesday 13rd July, 2011.

**Abstract**: The authors derived an a *posteriori* error estimator for *hp* adaptive discontinuous Galerkin (DG) methods for elliptic problems on 1–irregularly and isotropically refined, affine hexahedral meshes in three dimensions. The estimator yields upper and lower bounds for the error measured in terms of the natural energy norm. It is applied the estimate as an error indicator for energy norm error estimation in an *hp*–adaptive refinement algorithm. The numerical tests show that the indicator is efficient in locating and resolving isotropic corner singularities at exponential convergence rates.

**Key words and phrases**: Energy norm; a *posteriori*; *hp* adaptive discontinuous Galerkin methods; elliptic problems; three dimensions.

**Subject Classification**: 65N15; 65N30; 65N35; 65N50

## 1 what i have learned....

1. *aim:* is to develop the energy norm a *posteriori* error for the *hp* adaptive discontinuous Galerkin (DG) discretization for the following model in three dimensions:

$$-\Delta u(x) = f(x),\ x \in \Omega \subset \mathbb{R}^3, \qquad [1]$$

with the Dirichlet homogeneous boundary conditions:

$$u(x) = f(x),\ x \in \partial\Omega. \qquad [2]$$

2. *DG and hp*: according to the authors, working with discontinuous finite element spaces easily facilitates the use of variable polynomial degrees and local mesh refinement techniques on possibly irregularly refined meshes—the two key ingredients for $h_p$–adaptive algorithms

3. *some literature:* see [HOU 07, HOU 08, HOU 02, SCH 98, COC 00] .

4. *extension:* the article extends the two–dimensional analysis presented in [HOU 07] to 1–irregularly, isotropically refined affine hexahedral meshes in three dimensions.

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