# A brief Report on the article "Locally stabilized $P_1$–noconforming quadrilateral and hexahedral finite element methods for the Stokes equations "

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**Abstract**: This paper is concerned with a locally stabilized nonconforming finite element method for the stationary incompressible Stokes problems. The authors consider a locally stabilized finite element method and adopt the $P_1$–nonconforming quadrilateral and hexahedral elements for the approximation of both velocity and pressure variables. They invesigated numerically stabilized method based on an existing lowest equal–order nonconforming pair and the standard finite element method based on the same pair for the two and three dimensional Stokes equations. The article is a complement of some previous works in a sense that it demonstrates the high efficiency of the locally pressure-projection stabilized methods and illustates the flexibility of the defintion of pressure-projection operator. Optimal error estimates are derived in the energy nom and $L^2$–norm for the velocity and $L^2$–norm for the pressure.

**Key words and phrases**: Stokes equations; Stabilized $P_1$–noconforming quadrilateral and hexahedral finite element methods; Error estimates

**Subject Classification** : 65N30; 65N12; 76D07

## 1 Some knowledge

1. The paper is concerned with a locally stabilized nonconforming finite element method for the stationary incompressible Stokes problems.
2. It is known when the finite element method is applied to solve the Stokes equations, the finite element spaces for the velocity and pressure must satisfy the discrete inf-sup condition.
3. Several successful finite element spaces satifying the discrete inf-sup condition have been proposed and used. The article uder review presents a nice review on these finite element spaces satifying the discrete inf-sup condition

# References

[DOU 89] Dougals, J. and Wang, J.: An absolutely stabilized finite element method for the Stokes problem. *Math. Comput.*, **52**, 495–508 (1989).

[LI 08] Li, J. and He, Y.: A stabilized finite element method based on two local Gauss integration for the Stokes equations. *J. Comput. Appl. Math.*, **214**, 58–65 (2008).

[HE 08] He, Y. and Li, J.: A stabilized finite element method based on local polynomial pressure projection for the stationary Navier–Stokes equations. *Appl. Numer. Math.*, **58**, 1503–1514 (2008).

[GIR 86] Girault, V. and Raviart, P. A.: Finite Element Methods for Navier–Stokes equations. Theory and Algorithms. *Springer–Verlag, Berlin*, (1986).

[TEM 84] Temam, R.: Navier–Stokes equations: Theory and Numerical Analysis. *(Revised version) in: Studies in Mathematics and its Applications, Vol. 2, North Holland, Amsterdam*, (1984).