A brief Report on the article “Superconvergence of a stabilized finite element approximation for the Stokes equations using a local coarse mesh L^2 projection ”

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Numer. Methods Partial Differential Eq., 28, 115–126 (2012).

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Last update: Sunday 5th February, 2012; my hope I come back to this article to learn more

Abstract: The authors considered a low equal order finite element method to approximate the stationary Stokes equations in two dimensions. The method does not satisfy the known $inf - sup$ condition. This condition is required for instance to get the stability of the finite element methods approximating Stokes equations. A known stabilized form is then introduced and a well posedness for the new discrete problem is proved. The convergence order of the discrete velocity is h in H^1–norm whereas the convergence order of the discrete pressure is h in L^2–norm. Some postprocessed approximations are derived using the stated discrete velocity and the discrete pressure. These postprocessed approximations are the L^2 projections into piecewise polynomials of higher degree on some coarse meshes. It is proved that the convergence order of the stated postprocessed approximations is higher than that of the stated discrete velocity and the discrete pressure under some suitable choices for the piecewise polynomials of higher degree which are used to define these postprocessed approximations. They are then superconvergence results. Two assumptions are assumed to be satisfied in order to get the above stated results. The first one consists a regularity assumption on the domain on which the problem is posed. The second assumption consists a regularity assumption on the continuous mixed variational form.

Key words and phrases: Stokes equations; Stabilized finite element approximation; Superconvergence; coarse mesh; L^2 projection; Piecewise polynomials of higher degree

Subject Classification : 65N30

1 what I learned and some general remarks

1. the article is clear and useful
2. however, some feature quoted by the authors is not so clear, e.a. the authors said ”the superconvergence results have three prominent features. First, ... Second, they are derived on the basis of a large sparse, symmetric positive–definite systems of linear equations.
3. the construction of the spaces of piecewise polynomials of higher degree is not well clear

4. the reference [HEC] can be a useful source in order to do some numerical tests.

5. Sationary Stokes equations are

$$-\nu\,\Delta\,u(x) + \nabla\,p(x) = f(x),\ x \in \Omega, \quad [1]$$

$$\nabla \cdot u(x) = 0, \quad [2]$$

$$u(x) = 0, ,\ x \in \partial\Omega, \quad [3]$$

where Ω is a bounded domain in $\mathbb{R}^2$, $u(x) = (u_1(x), u_2(x))$ is the velocity vector, $p(x)$ is the pressure, and f is the prescribed body force, and $\nu > 0$ is the viscosity.

6. I think it is interesting to study te case of Nonsationary Stokes equations.

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[HEC] Hecht, F.; Pironneau, A.; Le Hyaric, and K. Ohtsuka: FreeFem ++ manual. http://www.freefem.org.