A BRIEF REPORT ON THE ARTICLE "SUPERCONVERGENCE FOR FINITE ELEMENT APPROXIMATION OF A CONVECTION-DIFFUSION EQUATION USING GRADED MESHES. "

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ABSTRACT. The article treats the superconvergence property of $\mathcal{Q}_1$ rectangular finite elements using a graded mesh for a model of the convection–diffusion problem. The graded mesh is an alternative to the Shishkin type and studied recently by the authors. It proved that, if u_h (where h is a parameter related to the definition of the mesh) is the finite element solution and u_I is the Lagrange interpolation of the exact solution u, $\| u_h - u_I\|_\varepsilon$ is of higher order than that of $\| u_h - u\|_\varepsilon$, where $\|\cdot\|_\varepsilon$ denotes to a weighted H^1-norm associated with the symmetric part of the differential equation. This stated superconvergence result with some existing interpolation error estimates yields an optimal–order convergence in L^2–norm. Both superconvergence in $\|\cdot\|_\varepsilon$-norm and optimal convergence order in L^2–norm stated in the before are almost optimal in the sense that the constants depend only on the logarithm of the singular perturbation parameter. These results have been obtained thanks to the combination of some known results concerning the superconvergence property and graded meshes. The article is useful and it deserves to be read.

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I really enjoyed reading this article: easy to read and the results are clear.
I fount it very useful and my hope I comeback to learn more from this article.

1. SOME BASIC KNOWLEDGE AND LITERATURE

1. some literature on the subject [2, 3, 4, 5]
2. The article under review deals with the approximation of a convection–diffusion problem by standard $\mathcal{Q}_1$ rectangular finite elements.
3. Because of the presence of boundary layers for convection-*dominated* problems, it is well known that the standard finite element elements produce poor approximations for these layers unless some extra–issues will be used , i.e. very fine meshes or appropriate meshes.
4. In the cases when the behavior of the exact solution is known, it is perhaps possible to design *a priori* adapted meshes which approximate the boundary layer well.
5. A lot of work has been to done to approximate convection-*dominated* problems. Probably, the most well-known approximations for this class of problems are those based on the use of the so-called Shishkin meshes. In particular, an optimal order of convergence has been proved when Shishkin meshes are used in combination with standard finite element methods or the so called streamline artificial diffusion methods, see for instance [5].
6. The article is an extension of a previous work [3].
7. Recently in the works [2, 3], it is used the so called graded meshes to approximate reaction-diffusion and convection–diffusion problems. The method of graded meshes was analysed in these two stated works and almost optimal estimates were obtained. On the other hand the so-called superconvergence property is studied using Shishkin meshes for convection–diffusion problems in, for instance, [6, 7]. By the way, superconvergence property is studied in first time (to our knowledge) by Zlamal [8]. The aim of this article is to analyse whether similar results to those obtained for Shishkin meshes are valid for graded meshes. This kind of mesh is an alternative to the Shishkin type:

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7.1 The authors prove superconvergence error estimates for the standard $\mathcal{Q}_1$ finite element approximation of model convection-diffusion problem when a graded mesh is used. Precisely, if u_h (where h is a parameter related to the definition of the mesh) is the finite element solution and u_I is the Lagrange interpolation of the exact solution u, it is proved that $\| u_h - u_I \|_\varepsilon$ is of higher order than that of $\| u_h - u \|_\varepsilon$, where $\| \cdot \|_\varepsilon$ denotes to a weighted H^1-norm associated with the symmetric part of the differential equation.

7.2 The stated superconvergence result in the previous item is combined with interpolation error estimates obtained in [3] to get an optimal–order convergence in L^2–norm.

7.3 Both superconvergence in $\| \cdot \|_\varepsilon$-norm and optimal convergence order in L^2–norm stated in the previous item are almost optimal in the sense that the constants depend only on the logarithm of the singular perturbation parameter.

7.4 The above results have been obtained thanks to combinations of the results of [3, 8]

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