## Some notes on the article "A priori error estimates for finite element methods with numerical quadrature for

nonmonotone nonlinear elliptic problems"

Abdulle, Assyr; Vilmart, Gilles Numer. Math. 121, No. 3, 397-431 (2012)

Report done by Professor Bradji, Abdallah Provisional home page: http://www.cmi.univ-mrs.fr/~bradji

Last update: Wednesday 19th September, 2012; sure I come back to this article to learn more...

Abstract: A quasilinear elliptic problem posed on a bounded polyhedron subset in  $\mathbb{R}^d$ , where  $d \leq 3$  is considered. The tensor involved in the quasilinear elliptic problem is assumed to be satisfied some suitable conditions which allow to get a unique solution. A family of partitions of the domain in simplicial or quadrilateral elements is introduced. A suitable quadrature formula is considered. Based on on this quadrature formulas, a finite element solution with numerical integration is defined. The existence of a finite element solution as well as some discrete a priori estimate are already known in the literature. Under some suitable conditions on the data of the problem, new error estimates in  $H^1$  and  $L^2$  norms are stated and proved. If in addition, the family of partitions is satisfying an inverse assumption, it is proved that the finite element solution is unique.

Key words and phrases: nonmonotone nonlinear elliptic problems; finite element methods; numerical quadrature; A priori error estimates

Subject Classification: 65N30; 65M60; 65D30

## some remarks...

- 1. literature:
  - (a) Continuous problem in practice [AMA 93]
  - (b) Finite element for nonmonotone problems [FEI 93, FEI 87]
  - (c) Well posedness for the continuous problem [CHI 09]

## References

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