A brief Report on the article [AND 10]“A Zienkiewicz-type finite element applied to fourth-order problems”

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Abstract: The authors consider the biharmonic equation and its associated eigenvalue problem. They present finite element schemes for both problems based on the use of a $\mathcal{C}^0$–nonconforming Zienkiewicz-type triangle elements. The convergence order of the both schemes is one in some discrete Sobolev of order two. It is presented a relatively simple postprocessing method that gives better accuracy for eigenvalues. It is based on a postprocessing technique whereby an additional solving of a source problem on augmented FE space is involved. Numerical examples explaining stated results are presented

Key words and phrases: fourth order equation; biharmonic equation; eigenvalues; finite element methods; nonconforming Zienkiewicz-type triangle elements

Subject classification:65N30; 65N25

1 Basic information, an overview, and some remarks

1. equation solved two main problems are solved, the domain Ω is a thin elastic plate :

   (a) equation solved:

   $$\Delta^2 u(x) = f(x),\ x \in \Omega, \qquad [1]$$

   with bounday condition

   $$u(x) = u_{\mathbf{n}}(x) = 0,\ x \in \partial\Omega. \qquad [2]$$

   (b) eigenvalue problem solved:

   $$\Delta^2 u(x) = \lambda\, u(x),\ x \in \Omega, \qquad [3]$$

   with bounday condition

   $$u(x) = u_{\mathbf{n}}(x) = 0,\ x \in \partial\Omega. \qquad [4]$$

2. weak formulation [1]–[2]: a weak for problem [1]–[2] is: find $u \in H_0^2(\Omega)$ such that

$$a(u,v) = \int_\Omega f(x)v(x)dx, \ \forall\, v \in H_0^2(\Omega), \qquad [5]$$

where

$$a(u,v) = \sum_{i,j=1}^{2} \int_\Omega u_{x_i x_j} u_{x_i x_j} dx, \qquad [6]$$

3. weak formulation [3]–[4]: a weak for problem [3]–[4] is: find $u \in H_0^2(\Omega)$ such that

$$a(u,v) = \lambda \int_\Omega u(x)v(x)dx, \ \forall\, v \in H_0^2(\Omega). \qquad [7]$$

2 Some finite element scheme and main results

The discretization is performed thanks to triangles. The degrees of freedom of an element K are chosen as:

$$p(a_i), \ i \in \{1,2,3\}. \qquad [8]$$

$$Dp(a_i)(a_j - a_i), \ (i,j) \in \{1,2,3\} \times \{1,2,3\} \text{ and } j \neq i. \qquad [9]$$

where $\{a_i; i \in \{1,2,3\}\}$ are the vertex of the triangle element.
The space can be chosen as

$$\mathcal{P}_K = \mathcal{P}_2 + \text{span}\{\lambda_i^2\lambda_j - \lambda_i\lambda_j^2; 1 \leq i < j \leq 3\}. \qquad [10]$$

It is proved that The degree of freedom are $\mathcal{P}_K$ unisolvent.
The finite element scheme suggested in the paper is a C^0 finite element.

1. convergence order for the biharmonic equation: using the interpolation error and second Strang Lemma, it is proved that the error is of order one in some discrete Sobolev of order two. The regularity required is $H^3(\Omega)$.

2. convergence order for the eigenvalue problem associated to the biharmonic equation: it is also proved that the approximate eigenfunction converges to its corresponding eigenfunction by order one some discrete Sobolev of order two.

3. postprocessing: it is presented a relatively simple postprocessing method that gives better accuracy for eigenvalues. It is based on a postprocessing technique whereby an additional solving of a source problem on augmented FE space is involved

References

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