A brief Report on the article [MIN 07]: "A new superconvergence of nonconforming rotated Q_1 element in 3D"

P. Ming, Z. Shi and Y. Xu

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Report done by Professor Bradji, Abdallah

Provisional home page: http://www.cmi.univ-mrs.fr/~bradji

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Abstract

The authors consider linear second order elliptic equations posed on a rectangular parallelepiped in three dimensions. They deal with a nonconforming finite element method using a nonconforming rotated Q_1 element. Several choices of the numerical integration which lead to optimal order in H^1 and L^2 as well as to superconvergence properties are provided. Numerical tests justifying theoretical results are presented.

1 Basic knowledge

Nonconforming rotated Q_1 element (NRQ1) is applied in many fields. In the present paper, (NRQ1) is used to approximate a second order elliptic equation in 3D. In addition to the optimal convergence order in H^1 and L^2, the authors prove that (NRQ1) generates superconvergence properties. The authors provide several methods of the numerical integrations which produce supconvergence properties NRQ1.

The equation to be solved:

$$-\sum_{i,j=1}^{3}\frac{\partial}{\partial x_i}\left(a_{ij}\frac{\partial}{\partial x_j}u(x)\right)=f(x),\ x\in\Omega, \qquad [1]$$

and

$$u(x)=0,\ x\in\partial\Omega. \qquad [2]$$

The coefficients $\{a_{ij}\}_{i,j\in\{1,3\}}$ satisfy :

- $a_{ij}\in\mathcal{W}^{3,\infty}(\Omega)$
- coercivity

$$\lambda\sum_{i=1}^{3}|\,\xi_i|^2\le\sum_{i,j=1}^{3}a_{ij}\xi_i\xi_j\le\mu\sum_{i=1}^{3}|\,\xi_i|^2. \qquad [3]$$

The subject of the first item allows us to get the required superconvergence properties, while the suject of the second item is a classical assumption in order to get the existence and uniqueness for problem [1]–[2].

Let $\mathcal{T}_h$ be a rectangular parallelepiped partition of Ω. For any $K\in\mathcal{T}_h$, let (x_0,y_0,z_0) be the element center and $2h_x$, $2h_y$, and $2h_z$ be the edge lengths in x, y, and z direction.

References

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