University of Annaba–Department of Technology
First year undergraduation **2009–2010**

Analysis

Supplementary problems

Complex numbers

Exercise 1. Study the convergence of the following serie:

$$\sum_{n=1}^{\infty} \frac{(1+i)^n}{5^{\frac{n}{2}}}.$$

Hint: use the fact that $1+i=\sqrt{2}\exp\left(i\frac{\pi}{4}\right)$

Exercise 2. Compute the convergence domain of: :

1.
$$\sum_{n=1}^{\infty} \frac{(z+2)^{n-1}}{(n+1)^3 4^n}, \tag{1}$$

2.
$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} z^{2n-1}}{(2n-1)!}, \tag{2}$$

3.
$$\sum_{n=1}^{\infty} n! z^n. \tag{3}$$

Exercise 3. Study the uniform convergence of the following series:

1.
$$\sum_{n=1}^{\infty} \frac{z^n}{n\sqrt{n+1}}, \ |z| \leq 1$$

2.
$$\sum_{n=1}^{\infty} \frac{1}{n^2+z^2}, \ 1 \leq |z| \leq 2,$$

3.
$$\sum_{n=1}^{\infty} \frac{\cos nz}{n^3}, \ |z| \leq 1,$$

4.
$$\sum_{n=1}^{\infty} \frac{\sqrt{n+1}}{n^2+|z|^2}$$

Exercise 4. Show that the following serie diverges:

$$\sum_{n=1}^{\infty} \frac{1}{n^{1+i}}$$

Exercise 5. Consider the following serie, which converges for $|z| \leq R$ to f:

$$f(z)=\sum_{n=1}^{\infty} a_n z^n.$$

Prove that

$$\frac{1}{2\pi}\int_0^{2\pi} |\, f(r\exp{(i\theta)}|^2 d\,\theta = \sum_{n=1}^{\infty} |\, a_n|^2 r^{2n}, \ \forall\, 0 \leq r \leq R.$$

Exercise 6. Study the convergence of the following series

1. $$\sum_{n=1}^{\infty} \frac{1}{n + |\, z|}.$$

2. $$\sum_{n=1}^{\infty} \frac{(-1)^n}{n + |\, z|}.$$

3. $$\sum_{n=1}^{\infty} \frac{1}{n^2 + |\, z|}.$$

4. $$\sum_{n=1}^{\infty} \frac{1}{n^2 + z}.$$