

Analysis
Supplementary problems
Complex numbers

Exercise 1. Compute the residus of each pole of the following functions:

1.

$$f(z) = \frac{z^2 - 2z}{(z + 1)^2(z^2 + 4)},$$

2.

$$f(z) = \frac{\exp z}{\sin^2 z},$$

3.

$$f(z) = \frac{2z + 1}{z^2 - z - 2},$$

4.

$$f(z) = \left(\frac{z + 1}{z - 1}\right)^2,$$

5.

$$f(z) = \frac{\sin z}{z^2},$$

6.

$$f(z) = \frac{1}{\operatorname{ch} z}.$$

Exercise 2. Compute the residue of the following function when $z = 0$:

$$f(z) = \frac{\cotgz \cothz}{z^3}.$$

Exercise 3. Compute the following integral:

$$\int_0^{\infty} \frac{dx}{x^6 + 1}$$

Exercise 4. Show that:

1.

$$\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2 + 1)^2(x^2 + 2x + 2)} = \frac{7\pi}{50}.$$

2.

$$\int_0^{2\pi} \frac{\cos(3\theta)}{5 - 4 \cos \theta} d\theta = \frac{\pi}{12}.$$

3.

$$\int_0^{2\pi} \frac{d\theta}{(5 - 3 \sin \theta)^2} = \frac{5\pi}{32}.$$

4.

$$\int_0^{\infty} \frac{\cos mx}{x^2 + 1} dx = \frac{\pi}{2} \exp(-m).$$

5.

$$\int_{\Gamma} \frac{\operatorname{ch} z}{z^3} dz = i\pi,$$

where Γ is the square of vertices $\{2, -2, 2i, -2i\}$.