University of Annaba–Department of Economy

First year undergraduation **2008–2009**

Analysis

Supplementary problems

Derivation

Exercise 1. Let us the consider the function $f(x) = |x|$. Recall that the function f is defined by

$$f(x) = x,\ x \geq 0 \text{ and } f(x) = -x,\ x \leq 0. \tag{1}$$

Recall that a function f is derivable on point x_0 if the following condition is fulfilled

$$\lim_{h\to 0} \frac{f(x_0+h) - f(x_0)}{h} \in \mathbb{R} \tag{2}$$

This means that f is derivable on point x_0 if the limit $\lim_{h\to 0} \frac{f(x_0+h) - f(x_0)}{h}$ exists (some times does not exist!!) and is finite. When the limit $\lim_{h\to 0} \frac{f(x_0+h) - f(x_0)}{h}$ exists and finite, we denote it by $f'(x_0)$.

- Using the definition (2), prove that for any $x_0 > 0$, f has derivative equal to 1 on x_0.
- Using the definition (2), prove that for any $x_0 < 0$, f has derivative equal to -1 on x_0.
- Using the definition (2), prove that for $x_0 = 0$, f is not derivable on x_0 (Hint: prove that $\lim_{h\to 0^+} \frac{f(x_0+h) - f(x_0)}{h} = 1$ and $\lim_{h\to 0^-} \frac{f(x_0+h) - f(x_0)}{h} = -1$.)

Exercise 2. Let us the consider the function f defined by

$$f(x) = x^2 - 1 + |x - 2|. \tag{3}$$

1. Using the definition of derivation stated in the previous exercise, prove that f is derivable on each $x_0 \neq 2$.
2. Prove that:

$$\frac{f(2+h) - f(2)}{h} = h + 4 + \frac{|h|}{h},\ \forall h \neq 0. \tag{4}$$

3. Using the (4), prove that

- $$\lim_{h\to 0^+} \frac{f(2+h) - f(2)}{h} = 5. \tag{5}$$

- $$\lim_{h\to 0^-}\frac{f(2+h)-f(2)}{h}=3. \tag{6}$$

4. Deduce then from the previous question that f is not derivable on $x_0=2$.

Exercise 3. Compute the derivative of the following functions:

$$f(x)=(x^2-5)(x^2-2x+7);\ g(x)=(\sqrt{x}+2x)(4x^2-1);\ h(x)=\frac{x^2+1}{5x-3};\ r(x)=\frac{x-\sqrt{x}}{5x^2-3};$$
$$l(x)=(\frac{1}{x}-3)\frac{x^2+3}{2x-3};\ m(x)=\sqrt{x}(2x-1)(x^3-x);\ n(x)=(x^3+4)^4$$

Exercise 4. Compute the derivative of the following functions:

$$f(x)=\sin(x)-\cos(x);\ g(x)=(\cos(x))(\cot(x));\ h(x)=x\cos(x)+x^2+1;\ r(x)=\tan(x)-\sin(x);$$
$$l(x)=\cos(\exp(x))+\exp(x)\cos(x);\ m(x)=\log(\cos(x));\ n(x)=\exp(-2x)\sin(3x)$$

Exercise 5.

Find the first derivative of the funtion $f(x)=x^x$

Exercise 6. Using the definition $f'(x_0)=\lim_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h}$ of the derivative of a function f on x_0, compute the derivative of f on x_0 in each of the following cases:

1. $f(x)=x$, $x_0=0$
2. $f(x)=x^2$, $x_0=1$
3. $f(x)=|x|$, $x_0=1$
4. $f(x)=|x|$, $x_0=0$ (Hint: use the fact that $|x|=x$, for $x\geq 0$ and $|x|=-x$, for $x\leq 0$.)

Exercise 7. Compute the n^{th} derivative of the following functions

$$f(x)=\sin(2x);\ f(x)=\cos(3x+5);\ h(x)=\frac{1}{x};\ r(x)=\frac{1}{1-x};\ l(x)=\frac{1}{1-x^2} \tag{7}$$

Exercise 8. Compute the first derivative of the following function

$$f(x)=\sqrt{1+\sqrt{1+\sqrt{x}}} \tag{8}$$

Exercise 9. Compute the first derivative f' of the function f defined by

$$f(x)=\frac{x^2+1}{x-1}. \tag{9}$$

Deduce then the derivative of the following functions:

$$g(x) = \frac{x^2+2x+2}{x};\ h(x) = \frac{x+1}{\sqrt{x}-1};\ l(x) = \frac{x^4+1}{x^2-1};\ k(x) = \frac{\sin^2 x+1}{\sin x-1};$$

$$u(x) = \sqrt{\frac{x^2+1}{x-1}};\ v(x) = \{\frac{x^2+1}{x-1}\}^2$$