

Analysis
Supplementary problems
Elementary Functions

Exercise 1. Simplify the following expression:

$$\mathcal{A} = (\exp(x) - \exp(-x))^2 - \exp(-x)(\exp(3x) + \exp(-x)) + \exp(-\log 3) + \exp(\log 2 - \log 3). \quad (1)$$

Exercise 2. Determine the smallest integer n such that:

$$1 - \left(\frac{4}{5}\right)^n \geq 0,97. \quad (2)$$

Exercise 3. Resolve in \mathbb{R} :

1.

$$\log \sqrt{2x-3} + \frac{1}{2} \log x = \log(6-x). \quad (3)$$

2.

$$\log(\log \exp(x) + \log \exp(\frac{1}{x})) = 1 - \log(x). \quad (4)$$

3.

$$\exp\left(\frac{3}{x}\right) - \exp(x+2) \geq 0. \quad (5)$$

4.

$$\log\left(\frac{2x+1}{x-1}\right) \leq 0. \quad (6)$$

5.

$$\log_a x > \log_a(3x+2). \quad (7)$$

Exercise 4. Resolve in $\mathbb{R} \times \mathbb{R}$:

$$\begin{cases} \exp(x) \times \exp(y) = \exp(30) \\ \log(x) + \log(y) = 3 \log(6) \end{cases} \quad (8)$$

Exercise 5. Determine the domain of definition of the following functions and compute their derivatives:

1.

$$f(x) = \log(x^2). \quad (9)$$

2.

$$f(x) = (\log(x))^2. \quad (10)$$

3.

$$f(x) = \exp(-\sin x). \quad (11)$$

4.

$$f(x) = \exp(x) \log x. \quad (12)$$

5.

$$f(x) = \frac{\log(x-1)}{2x+3}. \quad (13)$$