University of Annaba–"Departmant of Mathematics and Informatics MIAS"
Master 2 in Theoretical Physics **Wednesday, 25 January, 2012**

Exam 1

Analysis

Exercise 1. Can the following function be extended by continuity on $x_0 = 0$:

$$f(x) = \left(\frac{1+x}{1-x}\right)^{\ln(x)} \tag{1}$$

Exercise 2. Determine the constants α and β such that the function f given below is defferentiable on $\mathbb{R}$:

$$f(x) = \alpha\, x^2 + \beta\, x + 1,\ x \geq 0, \tag{2}$$

$$f(x) = \exp(x),\ x \leq 0. \tag{3}$$

Exercise 3. Study the continuity of the integer part function $f(x) = \mathrm{E}(x)$

Exercise 4. Consider the sequence

$$u_{n+1} = f(u_n),\ n \geq 0, \tag{4}$$

with

$$u_0 = a, \tag{5}$$

and

$$f(x) = \frac{(x-4)^2}{9}. \tag{6}$$

1. Compute $f([0,4])$ and $f([16,+\infty))$
2. Assume that $a > 16$
   (a) Show that the sequence u_n is increasing
   (b) Show that using the previous item that $\lim_{n\to+\infty} u_n = +\infty$.
3. What happens when $a = 16$?
4. Assume that $0 \leq a \leq 4$
   (a) Prove that $0 \leq u_n \leq 4$, for all n
   (b) Use the fact that $f(1) = 1$ to prove that

   $$|\,u_{n+1} - 1| \leq \frac{8}{9}|\,u_n - 1|. \tag{7}$$

   (c) Deduce from the previous item that $\lim_{n\to+\infty} u_n = 1$.