

Exam 1

Numerical Analysis

Exercise 1. Consider the following second order elliptic equation:

$$-u_{xx}(x) + u(x) = x^2 - 1, \quad x \in (0, 1) \quad (1)$$

with

$$u(0) = u(1) = 0. \quad (2)$$

Suggest two schemes one is a finite difference and the other is a finite volume. Study the convergence of these two schemes.

Exercise 2. Consider the following parabolic equation (with a given $T > 0$):

$$u_t(x, t) - u_{xx}(x, t) + 2u(x, t) = x - t, \quad (x, t) \in (0, 1) \times (0, T), \quad (3)$$

with

$$u(0) = u(1) = 0 \quad (4)$$

and

$$u(x, 0) = x. \quad (5)$$

Suggest two finite difference schemes, one is explicit and the other is implicit. Study the convergence of these two schemes.