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Dupire, G.; Boufflet, J.P.; Dambrine, M.; Villon, P.
On the necessity of Nitsche term
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MSC Classification: 65N30 65M15

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Review text:

The authors consider the Laplace equation with non homogeneous Dirichlet boundary conditions in one and two dimensions. A finite element scheme is suggested using a special variational formulation involving the boundary condition data as a term in the equation, whereas the space of the solution and the space of test functions are $\{u \in H^1(\Omega); \Delta\, u \in L^2(\Omega)\}$.

In the one dimensional case and using the above stated variational formulation, the authors suggested a piecewise linear space in order to approximate the exact solution and the space of the test functions. A convergence analysis based on the use of inf-sup inequality is provided. It is proved that the convergence order is optimal in the sense that the order is one, in the energy norm, and is two in the average norm.

Unfortunately, the analysis used in the one dimensional case can not be used in higher dimension for some geometrical reason stated by the authors. Consequently, some numerical experiments in dimension two are presented to justify the need of additional stabilization methods to obtain a reliable numerical method. The choice followed by the authors is the Nitsche method: that is to add some additional terms to the discrete bilinear and discrete linear forms. Thanks to some numerical experiments, it is proved that the addition of the Nitsche terms is not sufficient: a compatibility condition between the domain and the discrete grid should be added. Under this compatibility condition, discrete Nitsche problem is well posed and its solution converges to the exact solution by optimal order.

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