University of Annaba-Department of Mathematics and Informatics (MIAS)
First year undergraduation

## Analysis

## Remedial exam

Exercise 1. Study the derivability in $x_{0}=0$ of the function $f$ defined by

$$
\begin{equation*}
f(x)=x^{2} \sin \left(\frac{1}{x}\right), x \neq 0 \tag{1}
\end{equation*}
$$

and $f(0)=0$.
Exercise 2. Show the following iequality

$$
\begin{equation*}
\tan (x) \geq x, \quad \forall x \in\left[0, \frac{\pi}{2}\right] \tag{2}
\end{equation*}
$$

Exercise 3. Study the sequence sequence $u_{n}$ defined by $u_{1}=\sqrt{a}$ and

$$
\begin{equation*}
u_{n+1}=\sqrt{u_{n}+a}, n \in \mathbb{N} \backslash\{0,1\} \tag{3}
\end{equation*}
$$

where $a>0$.
Exercise 4. Compute the following limit:

$$
\begin{equation*}
\lim _{x \rightarrow 0} \frac{(1+\sin (x))^{\frac{1}{x}}-\exp \left(1-\frac{x}{2}\right)}{(1+\tan (x))^{\frac{1}{x}}-\exp \left(1-\frac{x}{2}\right)} \tag{4}
\end{equation*}
$$

Exercise 5. Can the following function be extended by continuity on $x_{0}=0$

$$
\begin{equation*}
f(x)=\left(\frac{1-x}{1+x}\right)^{\ln (x)} \tag{5}
\end{equation*}
$$

