

Analysis
Remedial exam

Exercise 1. Study the derivability in $x_0 = 0$ of the function f defined by

$$f(x) = x^2 \sin\left(\frac{1}{x}\right), \quad x \neq 0, \quad (1)$$

and $f(0) = 0$.

Exercise 2. Show the following inequality

$$\tan(x) \geq x, \quad \forall x \in \left[0, \frac{\pi}{2}\right]. \quad (2)$$

Exercise 3. Study the sequence sequence u_n defined by $u_1 = \sqrt{a}$ and

$$u_{n+1} = \sqrt{u_n + a}, \quad n \in \mathbb{N} \setminus \{0, 1\}, \quad (3)$$

where $a > 0$.

Exercise 4. Compute the following limit:

$$\lim_{x \rightarrow 0} \frac{(1 + \sin(x))^{\frac{1}{x}} - \exp\left(1 - \frac{x}{2}\right)}{(1 + \tan(x))^{\frac{1}{x}} - \exp\left(1 - \frac{x}{2}\right)}. \quad (4)$$

Exercise 5. Can the following function be extended by continuity on $x_0 = 0$

$$f(x) = \left(\frac{1-x}{1+x}\right)^{\ln(x)}. \quad (5)$$