Some remarks on the document presented by my students of Licence

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1 Introduction

The aim of this document is to provide my students of Licence with some remarks concerning the document presented by them since some weeks. I also put some other remarks concerning their work as well their motivation.

2 The aim of the work I suggested to them

As it is known in the departement, they asked me to supervise students from Licence. I suggested some work to be prepared by the students in the subject of *Finite Difference method*. In Licence, students do not take course in *Finite Difference method*. Therefore, preparing such work is not so obvious for them.

In parallel, I'm teaching a course of numerical analysis in the departement and then I'm interested to write some thing could be understood from people who have only 'modest' background in analysis. Thus, I have been preparing a course in numerical analysis, and as it is usual, I began with finite difference analysis. The course I'm preparing is written in English. My aim from the work I gave to my students is:

- concerning the subject itself
  - to understand what is *finite difference method*
  - what we need as material in order to approximate a given differential or partial equation
  - to apply finite difference method on some simple examples
  - to know the fameous Theorem in the convergence method: Stability and Concistency imply the convergence
- concerning learning some paths of *research*: I would like that my students be free to look, write,
  - how to use internet in order to search the convenient information

- to use e-mail to ask the information we want. For that reason, I gave to my students I'm supervising them my e-mail
- since students in general learn in French, then I would like that my students learn also in English.

3 Some remarks

3.1 Remarks on the document

- content,... The document they provided me is almost a translation of the document I have put in my home page from English to Franch. I would like that such document could be help in order to a better document using other references and other sources in internet.
- language,... the translation I could say that it is good with respect to the level of my students. Note that my students are in the third level of Licence in LMD in Algeria and the language by which learn students is either French or Arabic.
- document itself as hard paper,... it is some draft in which
  - the pages are not numbered
  - the pages are not ordered in the French writing direction, would say left to right
  - equations are not centered
  - from the mistakes I found in the document which are the same mistakes done the document I have put in my home page, I understood that students have not look to unserstand the mathematical side of the material put in my stated document
  - it remain
    * Title "Initiation à la méthode de differences finies"
    * Some introduction
    * to explain the aim of the document or the work
    * put the fameous Theorem of convergence: Concistency and Stability imply convergence
    * some example in which we explain how to apply the previous Theorem
    * to explain the plan of the document in the introduction
    * proof of the convergence of the finite difference scheme approximating the third example
    * correction of the convergence proof of the second example.

3.2 Remarks concerning work of my students

- efforts,... I think that the effort done by students is considerable:
  - translation from English to French is good
  - they came to discuss to me to Sidi Achour three times
- scientific,... I think that they have not tried yet to understand the mathematical material of finite difference method. This I think that this could be understood because of the limited time
- use internet they have used internet to translate from English to French: this is a good point
- e-mail even I gave to them my e-mail and I sent to my students two e-mails by they have not replied me ! I would like to use e-mail in order to know their questions or to fix appointments but they have not used e-mail.

4 I suggest the following introduction to be put in the document

Introduction of Godunov [GOD 77] as well as its first example, page 13–14.

It is also nice to put the plan of the document just after previous introduction, see my draft papers.

5 Convergence proof of second example

The convergence proof provided in the document is not correct. I have corrected it in my home page and here I'm correcting it in French as a help for my students.
En utilisant la formule de Talylor et le fait que $x_i = ih$ pour obtenir

$$\begin{aligned} u_i &= e^{i\left(\alpha h - \frac{\alpha^2 h^2}{2} + \alpha^2 h^2 \varepsilon_1(h)\right)} \\ &= e^{i\alpha h} e^{-x_i h \alpha^2 \left(\frac{1}{2} + \varepsilon_1(h)\right)} \\ &= e^{\alpha x_i} \left\{ 1 - x_i h \alpha^2 \left(\frac{1}{2} + \varepsilon_1(h)\right) + x_i h \alpha^2 \left(\frac{1}{2} + \varepsilon_1(h)\right) \varepsilon_2 \left(-x_i h \alpha^2 \left(\frac{1}{2} + \varepsilon_1(h)\right)\right)\right\} \\ &= u(x_i) + \mathcal{A}_h, \end{aligned} \quad [1]$$

avec

$$\mathcal{A}_h = -x_i h \alpha^2 \left(\frac{1}{2} + \varepsilon_1(h)\right) e^{\alpha x_i} \left\{ 1 - \varepsilon_2 \left(-x_i h \alpha^2 \left(\frac{1}{2} + \varepsilon_1(h)\right)\right)\right\}. \quad [2]$$

Nous avons utilisée les formules de Taylor suivantes:

$$\log(1+x) = x - \frac{x^2}{2} + x^2 \varepsilon(x), \quad [3]$$

et

$$e^x = 1 + x + x\varepsilon_2(x), \quad [4]$$

avec

$$\varepsilon_1(x) \to 0, \text{ et } \varepsilon_2(x) \to 0, \text{ quand } x \to 0. \quad [5]$$

Comme $\varepsilon_1(h) \to 0$, quand $h \to 0$ alors, pour h suffisamment petit, existe une constante positive C_1 indépendente de h telle que

$$|\varepsilon_1(h)| \leq C_1. \quad [6]$$

Comme $x_i \in [0,1]$, alors $-x_i h \alpha^2 \left(\frac{1}{2} + \varepsilon_1(h)\right) \to 0$, quand $h \to 0$. Cette derniere limite avec le fait que $\varepsilon_2(x) \to 0$ quand $x \to 0$ impliquent que $\varepsilon_2 \left(-x_i h \alpha^2 \left(\frac{1}{2} + \varepsilon_1(h)\right)\right) \to 0$, quand $h \to 0$. Donc, pour h suffisamment petit, existe une constante positive C_2 indépendente de h telle que

$$|\varepsilon_2 \left(-x_i h \alpha^2 \left(\frac{1}{2} + \varepsilon_1(h)\right)\right)| \leq C_2. \quad [7]$$

Ceci avec [6], [7], et le fait que $x_i \in [0,1]$, impliquent que, pour h pour h suffisamment petit

$$|\mathcal{A}_h| \leq C_3 h, \quad [8]$$

avec

$$C_3 = \alpha^2 \left(\frac{1}{2} + C_1\right)(1 + C_2) e^{\alpha}. \quad [9]$$

Ceci avec [1] impliquent, pour h suffisamment petit

$$|u_i - u(x_i)| \leq C_3 h, \ \forall i \in \{1, \ldots, N\} \quad [10]$$

References

[GOD 77] S. Godunov and V. Riabenki: Schémas aux Differences.*Editions Mir, Moscow, (French)*, 1977.

[SMI 85] G. D. Smith: Numerical Solution of Partial Differential Equations: Finite Difference Methods. *Oxford University Press, Third edition*, 1985.