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Mixed finite element methods for the fully nonlinear Monge-Ampère equation based on the vanishing moment method
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Review text:

The aim of the article is to provide a mixed finite element approximation for Monge–Ampère problem. The authors suggest a vanishing moment method which consist of a quasilinear fourth order problem with a singularly perturbed parameter $\varepsilon > 0$. It is first justified that for each $\varepsilon > 0$, the stated quasilinear fourth order problem has a unique solution u^ε. It is proved that u^ε converges uniformly towards the exact viscosity solution of Monge–Ampère problem, as $\varepsilon \to 0$. The quasilinear fourth order problem is equivalent to a nonlinear system of second–order equations. Using this system, the authors derive a weak mixed formulation to the quasilinear fourth order problem in which the solution is denoted by $(u^\varepsilon, \sigma^\varepsilon)$. Thanks to the use of Hermann–Miyoshi mixed elements, the authors suggest a finite element scheme in which the solution is denoted by $(u_h^\varepsilon, \sigma_h^\varepsilon)$. The existence, uniqueness, and the convergence $(u_h^\varepsilon, \sigma_h^\varepsilon)$ towards $(u^\varepsilon, \sigma^\varepsilon)$ is proved, using fixed point technique (since the discrete problem is nonlinear), under the assumption that the mesh parameter h is small enough and under a regularity assumption on $(u^\varepsilon, \sigma^\varepsilon)$. Finally, the authors present numerical tests showing the error estimates when the mesh parameter h is a power of ε.

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