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Finite-volume-element method for second-order quasilinear elliptic problems
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MSC Classification: 65M08 65M15

Keywords: Finite volume element method; second order quasilinear elliptic equations; error estimates

Review text:
The authors consider a general class of second order quasilinear equations posed on two dimensional polygonal convex domain. This class of equations includes in particular some known models like the equation of prescribed mean curvature, the subsonic flow on irrotational ideal compressible gas, and the Bratu's equation. The discretization is performed using a finite volume element method. A finite volume element scheme is presented. The existence and uniqueness is proved thanks to the introduction of a linearized operator which has a relation with the problem to be approximated. Several estimates have obtained and discussed. It is first proved that the convergence order is h in the energy norm. The convergence order is proved to be $h^{1-\frac{2}{r}}$ in $W^{1,\infty}$ when the exact solution is satisfying $u \in W^{2,r}(\Omega)$, where $2 < r \leq \infty$ but this last order can be improved to h when u is satisfying $u \in W^{2,\infty}(\Omega) \cap H^3(\Omega)$. It is also proved that the convergence order in L^∞ of the approximate solution is $h^2|\ln h|$. In the quadratic norm, it is proved that the convergence order is $h^2|\ln h|$ but this order can be improved to h^2 when the derivatives of some data are satisfying some condition which is satisfied by Bratu's equation. Numerical tests are presented to support the theoretical results. The article under review is nice and merits to be read.

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