Zentralblatt MATH Review Preview

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Sarkis, Marcus; Versieux, Henrique
Convergence analysis for the numerical boundary corrector for elliptic equations with rapidly oscillating coefficients
SIAM J. Numer. Anal. 46, No. 2, 545-576 (2008).

MSC Classification: 65N30 35B25 35B27 35B40 35B45 35B65 35C20

Keywords: elliptic equations; multiscaling; periodic equation; asymptotic expansion; corrector term; analytical approximation of the corrector term; a mixed finite element scheme; composite materials; asymptotic expansion; a priori error estimate; finite element scheme; a priori estimate in the finite element approximation

Review text:

The authors provide us with a finite element scheme for the elliptic problem $\mathcal{L}_\varepsilon u_\varepsilon = -\sum_{i,j=1}^{2} \frac{\partial}{\partial x_i}\left(a_{ij}\left(\frac{x}{\varepsilon}\right) \frac{\partial}{\partial x_j} u_\varepsilon(x)\right) = f(x)$ in Ω and $u_\varepsilon = 0$ on $\partial\Omega$, where $\Omega = (0,1)^2$, $a(y) = (a_{ij}(y))$ is a periodic symmetric positive definite matrix, and ε is a singular parameter such that $h >> \varepsilon$. To get a finite element scheme, the authors first derived a convenient asymptotic expansion for the exact solution u_ε. An a priori error estimate between the exact solution u_ε and this asymptotic expansion is proved under some weak regularity assumption on the exact solution. The previous stated asymptotic expansion contains a corrector term, to capture the homogeneous Dirichlet boundary condition of u_ε and denoted by θ_ε, is satisfying $\mathcal{L}_\varepsilon \theta_\varepsilon = 0$ and its boundary value is highly oscillatory. An analytical approximation for θ_ε is provided. A finite element scheme is suggested using some convenient finite element approximations for each term in the asymptotic expansion and the corrector term θ_ε. Depending on the regularity of the problem and of the functions included in the asymptotic expansion, an a priori error estimate of the suggested finite element scheme is proved. The order of the finite element scheme is $h^2 + \varepsilon^{3/2} + \varepsilon\, h$ on L^2–normi, and $h + \varepsilon^{1+\hat{\delta}}$ in the H_0^1–norm, for some $\hat{\delta} \in (-\frac{1}{4}, 0]$

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