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Lyashko, A.D.; Tayupov, Sh.I.; Timerbaev, M.R.
High-accuracy schemes of the finite element method for systems of degenerate elliptic equations on an interval
Russ. Math. 53, No. 7, 17-27 (2009).

MSC Classification: 65N30 65N15

Keywords: finite element method; degenerate elliptic equation; multiplicative singularity extraction; weighted Sobolev spaces

Review text:

This paper deals with a finite element scheme for a system of degenerate second order elliptic equations in one dimension. It is known that the exact solution in such problems have unbounded derivatives near degeneration points. Therefore, the discretization of such problems by the standard finite element method leads to loss of convergence of approximate solution to the exact one near degeneration points of coefficients of the differential operator of the problems. The finite element scheme suggested by the authors is based on a representation of the exact solution in terms of some known functions and unknown but smooth function. In the case of the Dirichlet problem with homogeneous boundary conditions, the authors prove that the exact solution is a product of two functions. The first function is very simple and describes the behavior of the exact solution near the the degeneration points of coefficients of the differential equations, and, the second function is treated as a new desired smooth function.

In the case of the Dirichlet problem with non homogeneous boundary conditions, the authors use the so called continuation function (of boundary values into the domain) to reduce the problem to a new problem with homogeneous Dirichlet boundary conditions. A construction for the continuation function is provided. By this way, the authors proved that the exact solution of the original problem can be represented as the sum of two terms. The first term is known and the second one is the product of a known function and a unknown smooth function.

In the both cases of homogeneous and non homogeneous boundary conditions and thanks to the above representation for the exact solution, the authors applied the usual finite element method to approximate the unknown smooth function which is the solution of a new system of smooth second order elliptic equations in one dimension. One remarks that this new unknown function is smooth, one can approximate it by piecewise polynomial function with a higher convergence order. This last finite element approximation allows us, thanks to the above stated representation, to obtain an approximation, for the exact solution of the original system, with optimal order in weighted Sobolev norms.

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