

## Algebra

## Supplementary problems

## Sequences

**Exercise 1.** List the first six terms of each of the following sequences.

$$x_n = \frac{n^2}{2^n}; x_n = (n, 1, -2n); x_n = (4, 0, 3); x_1 = x_2 = 1, x_{n+2} = x_{n+1} + x_n. \quad (1)$$

**Exercise 2.** Write the rule for the  $n$ -th term for the following arithmetic sequences

$$(7, 9, 11, 13, 15); (9, 5, 1, -3, -7); (3, 3.5, 4, 4.5, 5). \quad (2)$$

**Exercise 3.**

Write the rule for the  $n$ -th term of the arithmetic series with the following. (Use a system of equations.)

$$u_6 = -7, u_2 = 5; u_5 = 25, u_{30} = 150; u_{10} = -4, u_1 = 6. \quad (3)$$

**Exercise 4.** The terms of a sequence are given by  $u_n = 5 + 3(n - 1)$ .

1. Justify that the sequence  $(u_n)$  is arithmetic
2. State the first term and the common difference (reason)
3. Find the first five terms
4. Find the 20-th term
5. Which term would 302 be

**Exercise 5.** The maximum salary that a part time worker can expect to earn in a year is 7200 Euro. The starting salary is 5100 Euro and it is increased 75 per year. How long will it take a part time worker to earn the maximum available amount?

**Exercise 6.** The first term of an arithmetic sequence is 10 and the sixth term is 70.

1. Show that the general term is  $u_n = 12n - 2$

2. Find the first four terms and the twentieth term
3. Find the sum of the first ten terms

**Exercise 7.** The first four terms of a geometric sequence are 5, 15, 45, 135.

1. Find the common ratio
2. Find the tenth term
3. What is the sum of the first ten terms

**Exercise 8.** A principal of 2500 £ is invested with a return rate of 12 percent per annum (year). Assume that no money was ever taken out of this account and that the interest is compounded annually.

1. What will the earning (due to interest) be in the 7-th year since the principal was deposited?
2. How much will the investor have in the account after 7 years

**Exercise 9.** Evaluate:  $2 + 6 + 10 + 14 + \dots + 46$ .

**Exercise 10.** If  $3 - 6 + 12 - 24 + \dots + x = 63$ , then find  $x$ .

**Exercise 11.** Find the number of terms in the sequence  $27, -9, 3, -1, \dots, \frac{1}{27}$ .

**Exercise 12.**

How many odd numbers (beginning with 1) must be added before the total reaches one million?

**Exercise 13.** An investment, originally worth 1250 £ grow at the rate of 12 percent per year, compounded annually. Find

1. The value of the investment after five years.
2. The number of years that must pass before the investment is worth more than 10000 £

**Exercise 14.**

Two separate species of mice live in a National park.

The number of species  $\mathcal{A}$  was 12000 at the start of 1990 and increased by 200 per month after that date.

The number of species  $\mathcal{B}$  was 8000 at the start of 1990 and increased by 5 per month after that date.

1. If  $n$  is the number of months after the start of 1990 (when  $n = 0$ ), write a formulas for the populations of the two species  $\mathcal{A}_n$  and  $\mathcal{B}_n$ .
2. after how many months will the population of species  $\mathcal{B}$  exceed that of species  $\mathcal{A}$ ?