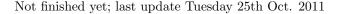
## University of Annaba–"M.I. Mathematiques et Informatique LMD" First year undergraduation

## Analyse

## **Complex numbers**



**Exercise 1.** (Third degree equation) Resolve the following third degree equation by remarking that  $z_0 = -i$  (recall that  $i^2 = -1$ ):

$$z^{3} - (16 - i)z^{2} + (89 - 16i)z + 89i = 0.$$
 (1)

**Exercice 2.** (Some computations) Show that if z = -1 - i, then  $z^2 + 2z + 2 = 0$ 

**Exercice 3.** (Complex and real forms)

1. Let z = x + iy and consider its complex conjugate  $\overline{z} = x - iy$ . Show that

$$x = \frac{z + \bar{z}}{2},\tag{2}$$

and

$$y = \frac{z - \bar{z}}{2i}.$$
(3)

2. Find the complex form, (2)–(3), using of the following equation x + y = 1.

**Exercice 4.** (Some computations) Simplify the following complex numbers

1.

$$z = (1+2i)(3-5i); \quad w = (4-3i)^2; \quad u = i^3; \quad v = i^4; \quad k = i^{31}$$
(4)

2.

$$z = \frac{1}{3-5i}; \ w = \frac{1+2i}{3-5i}; \ u = \frac{1}{(3-5i)^2}; \ v = \frac{1+2i}{1-\sqrt{3}i}.$$
 (5)

**Exercice 5.** (Some properties) Let  $z \in \mathbb{C} \setminus \{1\}$ . Prove by two methods that  $\frac{1+z}{1-z} \in i\mathbb{R}$  if and only if |z| = 1.

**Exercice 6.** (Some properties) Prove that

1. For all  $z, w \in \mathbb{C}$ :

$$|z+w|^{2} + |z-w|^{2} = 2(|z|^{2} + |w|^{2}).$$
(6)

2. For all  $z, w \in \mathbb{C}$ :

$$|z+w|^{2} \le (1+|z|^{2})(1+|w|^{2}).$$
(7)

3. In which case there is equality in (7)?

**Exercice 7.** (Some properties) Let  $z, w \in \mathbb{C}$  such that |z| < 1 and |w| < 1. Prove that

$$\left|\frac{z-w}{1-\bar{z}w}\right| < 1. \tag{8}$$

**Exercice 8.** (Polar form of complex number) Put in the polar form the following complex numbers:

$$z = i; \ w = -1 + i; \ u = -5 + i\sqrt{75}; \ v = \left(\frac{\sqrt{3} - i}{2}\right)^6; \ k = \left(\frac{2}{-\sqrt{3} + i}\right).$$
(9)

**Exercice 9.** (Square roots of a complex number) Find the square roots of the following complex numbers:

$$z = \frac{1}{2} \left( 1 - i\sqrt{3} \right); \quad w = \frac{1}{\sqrt{2}} \left( 1 + i \right). \tag{10}$$

**Exercice 10.** (Cubic roots of a complex number) Find the cubic roots of the complex number z = i and the fourth roots of the following complex number:

$$w = \frac{-1 - i\sqrt{2}}{2}.$$
 (11)

**Exercice 11.** (Moivre rule) Use the Moivre rule to prove the following expansion:

1.  

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta.$$
(12)  
2.  

$$\sin 2\theta = 2\sin \theta \cos \theta.$$
(13)  
3.  

$$\cos 3\theta = 4\cos^3 \theta - 3\cos \theta.$$
(14)

4. 
$$\sin 3\theta = -4\sin^3 \theta + 3\sin \theta. \tag{15}$$

**Exercice 12.** (Euler rule) Use the Euler rule to linearize the following expansion:

1.  

$$\cos^{4} \theta.$$
(16)  
2.  

$$\sin^{3} \theta \cos \theta.$$
(17)  
3.  

$$\sin^{4} \theta..$$
(18)  
4.  

$$\cos^{3} \theta \sin^{2} \theta.$$
(19)