University of Annaba–"Ecole Preparatoire aux Sciences et Techniques **2010–2011**

Analysis

Supplementary problems

Fourier series

Exercise 1. Provide the graph of the following functions:

1. Periode of f is $T = 2\pi$
$$f(x) = \begin{cases} \sin x, & 0 \leq x \leq \pi \\ 0, & \pi < x < 2\pi. \end{cases} \tag{1}$$
2. Periode of f is $T = 4$
$$f(x) = \begin{cases} -2, & -2 \leq x \leq 0 \\ 1, & 0 \leq x < 2. \end{cases} \tag{2}$$
3. Periode of f is $T = \pi$
$$f(x) = \operatorname{tg}(x), \ -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}. \tag{3}$$

Exercise 2. Let f be the following function of periode $T = 10$:

$$f(x) = \begin{cases} 0, & -5 \leq x \leq 0 \\ 3, & 0 < x < 5. \end{cases} \tag{4}$$

1. Provide the graph of $f(x)$,
2. Compute the Fourier serie of deduce an entier serie of $f(x)$
3. Use Jordan's Theorem to study the convergence of the Fourier serie.
4. How to chose the values of f in $\{10k, 5 + 10k; k \in \mathbb{Z}\}$ such that Fourier serie corresponding to the new function converges to this new function.

Exercise 3. Let $f(x)$ be a periodic function of periode $T = 2\pi$ and

$$f(x) = x^2 - \pi^2, \ x \in [-\pi, \pi[\tag{5}$$

1. For which x, f can be represented by a Fourier serie
2. Compute the Fourier serie of $f(x)$
3. Use Parseval equality to compute the sum $\sum_{n=1}^{\infty} \frac{1}{n^4}$

Exercise 4. Let $f(x)$ be the function defined by

$$f(x) = 1, \ x \in [0, \pi[. \tag{6}$$

1. How to chose $f(x)$ such that Fourier serie of $f(x)$ contains only $\sin x$,
2. compute this Fourier serie.