University of Annaba–Department of Technology

Second year undergraduation **2009–2010**

Analysis

Supplementary problems

Series

Exercise 1. Explain the following equality:

$$\sum_{n\geq m} u_n = \sum_{n=m}^{+\infty} u_n, \qquad [1]$$

where $m \in \mathbb{N}$ is a given integer

Exercise 2. Let $m \in \mathbb{N}$ is a given integer. Explain why that the convergece of the series $\sum_{n\geq 0} u_n$ is equivalent to the convergente of $\sum_{n\geq m} u_n$.
More precise, explain that if $\sum_{n\geq 0} u_n$ is convergente, then the following identity holds:

$$\sum_{n\geq 0} u_n = \sum_{n\geq m} u_n - \sum_{n=0}^{m-1} u_n. \qquad [2]$$

Exercise 3.

1. Let $m \in \mathbb{N}$ be a given integer. Compute the following sum
$$\sum_{n=0}^{m} \left(\sqrt{n+1} - \sqrt{n}\right). \qquad [3]$$
2. Note that
$$\sum_{n=0}^{\infty} \left(\sqrt{n+1} - \sqrt{n}\right) = \lim_{m\to\infty} \sum_{n=0}^{m} \left(\sqrt{n+1} - \sqrt{n}\right), \qquad [4]$$
prove that, using the first item of this exercise
$$\sum_{n=0}^{\infty} \left(\sqrt{n+1} - \sqrt{n}\right) = +\infty. \qquad [5]$$
3. what we could deduce?

Exercise 4. In the following exercises, determine the convergence, divergence, absolute convergence of the given series using any test (criteria) and give reasons:

1. $$\sum_{n\geq 1} \frac{n^2}{n!}. \qquad [6]$$
2. $$\sum_{n\geq 1} (-1)^{n-1} \frac{n^2}{n!}. \qquad [7]$$
3. $$\sum_{n\geq 1} \frac{n^n}{n!}. \qquad [8]$$

4.

$$\sum_{n\geq 1}(-1)^{n-1}\frac{n}{3^n}. \qquad [9]$$

5.

$$\sum_{n\geq 1}(-1)^{n-1}\frac{n}{3^n}. \qquad [10]$$

6.

$$\sum_{n\geq 1}\frac{(n+1)!}{2^n\, n!}. \qquad [11]$$

7.

$$\sum_{n\geq 1}\frac{n^n}{(n!)^2}. \qquad [12]$$

8.

$$\sum_{n\geq 1}(-1)^{n-1}\frac{6}{n(\log\, n)^2}. \qquad [13]$$

9.

$$\sum_{n\geq 1}\frac{4n}{1+n^2}. \qquad [14]$$

10.

$$\sum_{n\geq 1}\frac{2^n}{n\, 3^n}. \qquad [15]$$

Exercise 5. Study the convergence series

- Criteria of comparaison

$$\sum_{n\geq 1}\frac{\log\, n}{2n^3}. \qquad [16]$$

- Criteria of Alembert (Ratio)

$$\sum_{n\geq 1}\frac{2^n\, n}{n^n} \text{ and } \sum_{n\geq 1}\frac{k^n}{n^k}. \qquad [17]$$

- Criteria of Cauchy (Sqrt)

$$\sum_{n\geq 1}\left(\frac{n+a}{n+b}\right)^{n^2}. \qquad [18]$$

- Criteria of Integral

$$\sum_{n\geq 1}\frac{\log\, n}{n}. \qquad [19]$$

Exercise 6. Study the following series

- $$\sum_{n\geq 1}\left(\frac{1}{3^n}+\frac{1}{n^2+1}\right). \qquad [20]$$

- $$\sum_{n\geq 1} n\log\left(\frac{n+2}{n+1}\right). \qquad [21]$$

- $$\sum_{n\geq 1}\frac{(-1)^n}{\log\, n}. \qquad [22]$$

Exercise 7. Study the absolute convergence of the following series

- $$\sum_{n\geq 1}\frac{(-1)^n}{\sqrt{n}}.$$ [23]

- $$\sum_{n\geq 1}\frac{\sin n}{3^n}.$$ [24]

- $$\sum_{n\geq 2}\frac{(-1)^n}{\log n}.$$ [25]

Exercise 8. Prove that there exist constants α, β, and γ such that, for all n

$$\frac{1}{n(n+1)(n+2)}=\frac{\alpha}{n}+\frac{\beta}{n+1}+\frac{\gamma}{n+2}.$$ [26]

Deduce then the sum $\sum_{n=1}^{\infty}\frac{1}{n(n+1)(n+2)}$

Exercise 9. Prove that the following identity holds, for all n

$$\operatorname{arctg}\frac{1}{2n^2}=\operatorname{arctg}\frac{1}{2n-1}-\operatorname{arctg}\frac{1}{2n+1}.$$ [27]

Deduce then the sum $\sum_{n=1}^{\infty}\operatorname{arctg}\frac{1}{2n^2}$