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University of Annaba-Department of Technology
Second year undergraduation
    Analysis
    Supplementary problems
    Series
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2009-2010

Exercise 1. Explain the following equality:

$$
\begin{equation*}
\sum_{n \geq m} u_{n}=\sum_{n=m}^{+\infty} u_{n} \tag{1}
\end{equation*}
$$

where $m \in \mathbb{N}$ is a given integer
Exercise 2. Let $m \in \mathbb{N}$ is a given integer. Explain why that the convergece of the series $\sum_{n \geq 0} u_{n}$ is equivalent to the convergente of $\sum_{n \geq m} u_{n}$.
More precise, explain that if $\sum_{n \geq 0} u_{n}$ is convergente, then the following identity holds:

$$
\begin{equation*}
\sum_{n \geq 0} u_{n}=\sum_{n \geq m} u_{n}-\sum_{n=0}^{m-1} u_{n} \tag{2}
\end{equation*}
$$

## Exercise 3.

1. Let $m \in \mathbb{N}$ be a given integer. Compute the following sum

$$
\begin{equation*}
\sum_{n=0}^{m}(\sqrt{n+1}-\sqrt{n}) \tag{3}
\end{equation*}
$$

2. Note that

$$
\begin{equation*}
\sum_{n=0}^{\infty}(\sqrt{n+1}-\sqrt{n})=\lim _{m \rightarrow \infty} \sum_{n=0}^{m}(\sqrt{n+1}-\sqrt{n}) \tag{4}
\end{equation*}
$$

prove that, using the first item of this exercise

$$
\begin{equation*}
\sum_{n=0}^{\infty}(\sqrt{n+1}-\sqrt{n})=+\infty \tag{5}
\end{equation*}
$$

3. what we could deduce?

Exercise 4. In the following exercises, determine the convergence, divergence, absolute convergence of the given series using any test (criteria) and give reasons:
1.

$$
\begin{equation*}
\sum_{n \geq 1} \frac{n^{2}}{n!} \tag{6}
\end{equation*}
$$

2. 

$$
\begin{equation*}
\sum_{n \geq 1}(-1)^{n-1} \frac{n^{2}}{n!} \tag{7}
\end{equation*}
$$

3. 

$$
\begin{equation*}
\sum_{n \geq 1} \frac{n^{n}}{n!} \tag{8}
\end{equation*}
$$

4. 

$$
\begin{equation*}
\sum_{n \geq 1}(-1)^{n-1} \frac{n}{3^{n}} . \tag{9}
\end{equation*}
$$

5. 

$$
\begin{equation*}
\sum_{n \geq 1}(-1)^{n-1} \frac{n}{3^{n}} \tag{10}
\end{equation*}
$$

6. 

$$
\begin{equation*}
\sum_{n \geq 1} \frac{(n+1)!}{2^{n} n!} \tag{11}
\end{equation*}
$$

7. 

$$
\begin{equation*}
\sum_{n \geq 1} \frac{n^{n}}{(n!)^{2}} \tag{12}
\end{equation*}
$$

8. 

$$
\begin{equation*}
\sum_{n \geq 1}(-1)^{n-1} \frac{6}{n(\log n)^{2}} \tag{13}
\end{equation*}
$$

9. 

$$
\begin{equation*}
\sum_{n \geq 1} \frac{4 n}{1+n^{2}} \tag{14}
\end{equation*}
$$

10. 

$$
\begin{equation*}
\sum_{n \geq 1} \frac{2^{n}}{n 3^{n}} \tag{15}
\end{equation*}
$$

Exercise 5. Study the convergence series

- Criteria of comparaison

$$
\begin{equation*}
\sum_{n \geq 1} \frac{\log n}{2 n^{3}} \tag{16}
\end{equation*}
$$

- Criteria of Alembert (Ratio)

$$
\begin{equation*}
\sum_{n \geq 1} \frac{2^{n} n}{n^{n}} \text { and } \sum_{n \geq 1} \frac{k^{n}}{n^{k}} \tag{17}
\end{equation*}
$$

- Criteria of Cauchy (Sqrt)

$$
\begin{equation*}
\sum_{n \geq 1}\left(\frac{n+a}{n+b}\right)^{n^{2}} \tag{18}
\end{equation*}
$$

- Criteria of Integral

$$
\begin{equation*}
\sum_{n \geq 1} \frac{\log n}{n} \tag{19}
\end{equation*}
$$

Exercise 6. Study the following series
-

$$
\begin{equation*}
\sum_{n \geq 1}\left(\frac{1}{3^{n}}+\frac{1}{n^{2}+1}\right) \tag{20}
\end{equation*}
$$

$$
\begin{equation*}
\sum_{n \geq 1} n \log \left(\frac{n+2}{n+1}\right) \tag{21}
\end{equation*}
$$

- 

$$
\begin{equation*}
\sum_{n \geq 1} \frac{(-1)^{n}}{\log n} \tag{22}
\end{equation*}
$$

Exercise 7. Study the absolute convergence of the following series

$$
\begin{equation*}
\sum_{n \geq 1} \frac{(-1)^{n}}{\sqrt{n}} \tag{23}
\end{equation*}
$$

- 

$$
\begin{equation*}
\sum_{n \geq 1} \frac{\sin n}{3^{n}} \tag{24}
\end{equation*}
$$

- 

$$
\begin{equation*}
\sum_{n \geq 2} \frac{(-1)^{n}}{\log n} \tag{25}
\end{equation*}
$$

Exercise 8. Prove that there exist constants $\alpha, \beta$, and $\gamma$ such that, for all $n$

$$
\begin{equation*}
\frac{1}{n(n+1)(n+2)}=\frac{\alpha}{n}+\frac{\beta}{n+1}+\frac{\gamma}{n+2} . \tag{26}
\end{equation*}
$$

Deduce then the sum $\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$
Exercise 9. Prove that the following identity holds, for all $n$

$$
\begin{equation*}
\operatorname{arctg} \frac{1}{2 n^{2}}=\operatorname{arctg} \frac{1}{2 n-1}-\operatorname{arctg} \frac{1}{2 n+1} . \tag{27}
\end{equation*}
$$

Deduce then the sum $\sum_{n=1}^{\infty} \operatorname{arctg} \frac{1}{2 n^{2}}$

