University of Annaba–Department of Technology

Second year undergraduation **2009–2010**

Analysis

Supplementary problems

Series

Exercise 1. Study the convergence of the following serie when $x = 0$:

$$\frac{4-x}{7x+2} + \frac{1}{3}\left(\frac{4-x}{7x+2}\right)^2 + \frac{1}{5}\left(\frac{4-x}{7x+2}\right)^3. \quad [1]$$

Exercise 2. Find the domain of convergence of the following series:

1. $$\sum_{n\geq 1} \frac{nx}{1+nx}. \quad [2]$$
2. $$\sum_{n\geq 1} \exp -nx. \quad [3]$$
3. $$\sum_{n\geq 0} \frac{x^2}{(1+x^2)^n}. \quad [4]$$
4. $$\sum_{n\geq 1} \frac{(-1)^n}{(x+3)^n}. \quad [5]$$

Exercise 3.

1. Study the uniform convergence of the following serie on the interval $[-2,-1]$:
$$\sum_{n\geq 1} \frac{x}{(1+(n-1)\,x)\,(1+nx)}. \quad [6]$$
2. Use the Weierstrass criteria to study the uniform convergence of
   - $$\sum_{n\geq 1} \frac{x^n}{n}. \quad [7]$$
   - $$\sum_{n\geq 1} \frac{\cos(nx+1)}{n^2+1}. \quad [8]$$

Exercise 4. Compute the sum of the following series:

1. $$\sum_{n\geq 1} nx^{n-1}. \quad [9]$$
2. $$\sum_{n\geq 1} \frac{x^n}{n}. \quad [10]$$