University of Annaba–Department of Technology

Second year undergraduation **2009–2010**

Analysis

Supplementary problems

Series

Exercise 1. Determine the convergence domain of the following series:

1. $$\sum_{n\geq 1}(-3)^n\frac{x^n}{\sqrt{n+1}}. \tag{1}$$

2. $$\sum_{n\geq 1}\frac{n\,(x+2)^n}{3^{n+1}}. \tag{2}$$

3. $$\sum_{n\geq 0}(-1)^n\frac{x^{2n}}{2^{2n}\,(n!)^2}. \tag{3}$$

4. $$\sum_{n\geq 1}n^n\,(x+3)^n\,. \tag{4}$$

Exercise 2. We know that:

$$\frac{1}{1-x}=1+x+x^2+\ldots=\sum_{n\geq 0}x^n. \tag{5}$$

1. Use [5] to find an entier serie for $\frac{1}{2+x}$,
2. deduce an entier serie for $\frac{x^3}{2+x}$
3. Use [5] to find an entier serie for $\frac{1}{(1-x)^2}$, and determine the domain of convergence.

Exercise 3. Compute in the MacLaurin series of the following functions and determine the domain of convergence of these series:

1. $f(x)=\arctan x$
2. $f(x)=\log(1+x)$

Exercise 4.

1. Determine the Taylor serie of $f(x)=\exp x$ with $x_0=-2$.
2. Compute in the MacLaurin series of the function $f(x)=\cos x$ and show that MacLaurin serie converges to $f(x)=\cos x$.
3. Deduce from the previous item the entier serie of $f(x)=\sin x$