University of Annaba–Department of Economy
First year undergraduation **2008–2009**

Algebra

Supplementary problems

Sets theory

Exercise 1. Let $\Xi = \{1,2,3,4,5,6,7\}$ be the universal set, and consider the following subsets of Ξ:

$$\mathcal{A} = \{1,2,3\},\ \mathcal{B} = \{2,4,6\},\ \mathcal{C} = \{1,3,5,7\}.$$

Compute the folowing sets:

1. $\mathcal{C}_{\Xi}^{\mathcal{A}}$, $\mathcal{C}_{\Xi}^{\mathcal{B}}$, $\mathcal{C}_{\Xi}^{\mathcal{C}}$.
2. $\mathcal{A} \cup \mathcal{B}$, $\mathcal{A} \cup \mathcal{C}$, $\mathcal{B} \cup \mathcal{C}$.
3. $\mathcal{A} \cap \mathcal{B}$, $\mathcal{A} \cap \mathcal{C}$, $\mathcal{B} \cap \mathcal{C}$.
4. $\mathcal{A} \times \mathcal{A}$, $\mathcal{A} \times \mathcal{B}$, $\mathcal{A} \times \mathcal{B} \times \mathcal{C}$.

Exercise 2.

Let $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ be three sets. Show that

1. $\mathcal{A} \cup (\mathcal{B} \cap \mathcal{C}) = (\mathcal{A} \cup \mathcal{B}) \cap (\mathcal{A} \cup \mathcal{C})$.
2. $\mathcal{A} \times (\mathcal{B} \cap \mathcal{C}) = (\mathcal{A} \times \mathcal{B}) \cap (\mathcal{A} \times \mathcal{C})$.
3. $\mathcal{A} \times (\mathcal{B} \cup \mathcal{C}) = (\mathcal{A} \times \mathcal{B}) \cup (\mathcal{A} \times \mathcal{C})$.

Exercise 3. Let $\mathcal{A}$ be a set of n elements. How many element are in each of the following sets:

1. $\mathcal{A} \times \mathcal{A}$.
2. $\{(x,y) \in \mathcal{A} \times \mathcal{A},\ x \neq y\}$.
3. $\mathcal{A} \times \mathcal{A} \times \mathcal{A}$.
4. $\{(x,y,z) \in \mathcal{A} \times \mathcal{A} \times \mathcal{A},\ x \neq y,\ x \neq z,\ y \neq z\}$.

Exercise 4. Let $\mathcal{X}\rangle$ be the universal set consisting of all people of the university of Annaba Consider the following subsets:

1. $\mathcal{M}$ = the set of all males,
2. $\mathcal{C}$ = the set of all college students,

3. $\mathcal{I} =$ the set of all intelligent people,
4. $\mathcal{F} =$ the set of all females,
5. $\mathcal{S} =$ the set of all smoking people,
6. $\mathcal{P} =$ the set of all Professors,
7. $\mathcal{W} =$ the set of all well–dressed people.

Translate each of the following sentences into an equation or an equality using only the letters standing for sets and the symbols $=$, $\emptyset$, $\mathcal{C}$ (complementary set), $\cap$, $\cup$. (For example, the sentence "All college students are intelligent" means that the set of college students is a subset of the set of the set of intelligent people, i.e., $\mathcal{C} \subset \mathcal{I}$. But we are not permitted to use the symbol $\subset$. Hence we replace $\mathcal{C} \subset \mathcal{I}$ with $\mathcal{C} \cap \mathcal{I} = \mathcal{C}$, $\mathcal{C} \cup \mathcal{I} = \mathcal{I}$, or $\mathcal{C} \cap \mathcal{C}^{\mathcal{I}}_{\Xi} = \emptyset$. Similarily, the sentence " Some college students are intelligent" means that there is at least one member of the insection $\mathcal{C} \cap \mathcal{I}$. Hence this sentence is translated into $\mathcal{C} \cap \mathcal{I} \neq \emptyset$.

1. All professors are smoking,
2. No male college student is well–dressed,
3. Some professors are smoking,
4. Some smoking professors are neither intelligent nor well–dressed,
5. college students and professors are smoking,
6. If a person is smoking, then that person is intelligent,
7. A person is smoking if and only if he is intelligent.

Exercise 5. Which of the following are correct and why

1. $\{1\} \in \{\{1\}\}$
2. $\{1\} \in \{1, \{1\}\}$
3. $\{1\} \subset \{\{1\}\}$
4. $\{1\} \subset \{1, \{1\}\}$

Exercise 6. Give an example of two sets $\mathcal{A}$ and $\mathcal{B}$ such that both $\mathcal{A} \in \mathcal{B}$ and $\mathcal{A} \subset \mathcal{B}$ are true.