Relaxation and Numerical Approximation of a Two-Fluid Two-Pressure Model

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Context and Outline

This work fall within the scope of a Joint Research Group LJLL-CEA Saclay on multiphase flows and coupling of multiscale models:
LJLL: C. Chalons, F. Coquel, E. Godlewsky, F. Lagoutière, N. Seguin, P.-A. Raviart,
CEA Saclay: A. Ambroso, B. Boutin,T. Galié,
+ some-time participation from EDF: J.-M. Hérard, O. Hurisse.

Motivation: Nuclear reactor cooling and neutron moderation $\longrightarrow$ need for stable computations of liquid water-vapor flows.

Outline

- Two-Fluid Two-Pressure Model (equilibrium model)
- Relaxation method
- Riemann solver for the Relaxation Model
- Some numerical results
- Conclusions and Perspectives

The Two-Fluid Two-Pressure Model

$$
\begin{cases}
\partial_t \alpha_1 + u_I \partial_x \alpha_1 = 0 \\
\partial_t \alpha_1 \rho_1 + \partial_x \alpha_1 \rho_1 u_1 = 0 \\
\partial_t (\alpha_1 \rho_1 u_1) + \partial_x (\alpha_1 \rho_1 u_1^2 + \alpha_1 p_1(\rho_1)) - p_I \partial_x \alpha_1 = 0 \\
\partial_t \alpha_2 \rho_2 + \partial_x \alpha_2 \rho_2 u_2 = 0 \\
\partial_t (\alpha_2 \rho_2 u_2) + \partial_x (\alpha_2 \rho_2 u_2^2 + \alpha_2 p_2(\rho_2)) + p_I \partial_x \alpha_1 = 0
\end{cases}
$$

$$\alpha_1 + \alpha_2 = 1 \, .$$

$\longrightarrow$ Interface velocity and interface pressure are chosen as

$$u_I = u_2, \quad p_I = p_1$$

Consequence: the void fraction is transported by a pure contact discontinuity.

(see M.R. Baer and J.W. Nunziato, (1986), T. Gallouët, J.M. Hérard and N. Seguin, (2004), P. Embid and M. Baer, (1992)).

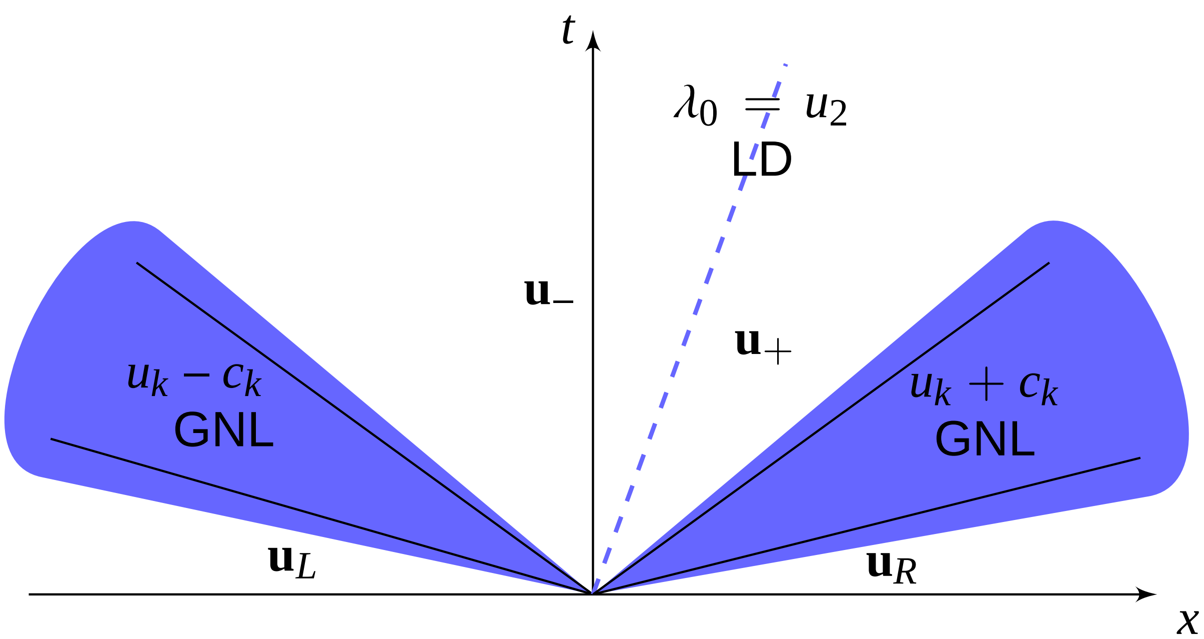
Properties of the model

Five **real** eigenvalues

$$\lambda_1 = u_1 - c_1, \quad \lambda_0 = u_2, \quad \lambda_2 = u_1 + c_1,$$
$$\lambda_3 = u_2 - c_2, \quad \lambda_4 = u_2 + c_2.$$

Hyperbolic if $u_2 \neq u_1 \pm c_1$.

The characteristic field associated to λ_0 is **LD** and the characteristic fields associated to $\{\lambda_i\}_{i=1,\dots,4}$ are **GNL**.



Relaxation system

$$\begin{cases} \partial_t \alpha_1 + u_2 \partial_x \alpha_1 = 0 \\ \partial_t(\alpha_1\rho_1) + \partial_x(\alpha_1\rho_1 u_1) = 0 \\ \partial_t(\alpha_1\rho_1 u_1) + \partial_x(\alpha_1\rho_1 u_1^2 + \alpha_1 \Pi_1) - \Pi_1 \partial_x \alpha_1 = 0 \\ \partial_t(\alpha_2\rho_2) + \partial_x(\alpha_2\rho_2 u_2) = 0 \\ \partial_t(\alpha_2\rho_2 u_2) + \partial_x(\alpha_2\rho_2 u_2^2 + \alpha_2 \Pi_2) + \Pi_1 \partial_x \alpha_1 = 0 \\ \partial_t \mathcal{T}_1 + u_2 \partial_x \mathcal{T}_1 = \lambda(\tau_1 - \mathcal{T}_1) \\ \partial_t \mathcal{T}_2 + u_2 \partial_x \mathcal{T}_2 = \lambda(\tau_2 - \mathcal{T}_2) \end{cases}$$

with $a_k > \rho_k c_k,\ k = 1, 2$ (Whitham stability condition) and

$$\Pi_k = p_k(1/\mathcal{T}_k) + a_k^2(\mathcal{T}_k - \tau_k),\ k = 1, 2.$$

We will use the following short notations

$$\partial_t \mathbf{v} + \partial_x \mathbf{g}(\mathbf{v}) + \mathbf{d}(\mathbf{v})\partial_x \mathbf{v} = \lambda \mathcal{R}(\mathbf{v}),\ t > 0,\ x \in \mathbb{R}.$$

Formally, when $\lambda \to \infty$ this system converges to the previous (equilibrium) two-fluid model.

(see S. Jin and Z. Xin, (1995), F. Coquel, E. Godlewski, A. In, B. Perthame and P. Rascle, (2001), C. Chalons and F. Coquel, (2005)).

Splitting

We solve the relaxation system by a splitting procedure:

- First step: we solve $\partial_t \mathbf{v} + \partial_x \mathbf{g}(\mathbf{v}) + \mathbf{d}(\mathbf{v})\partial_x \mathbf{v} = 0$
- Second step: we solve $\partial_t \mathbf{v} = \lambda \mathcal{R}(\mathbf{v})$ with $\lambda \to \infty$

In the following we will focus on the first step and, in particular on the Riemann solver needed to implement a Godunov type method.

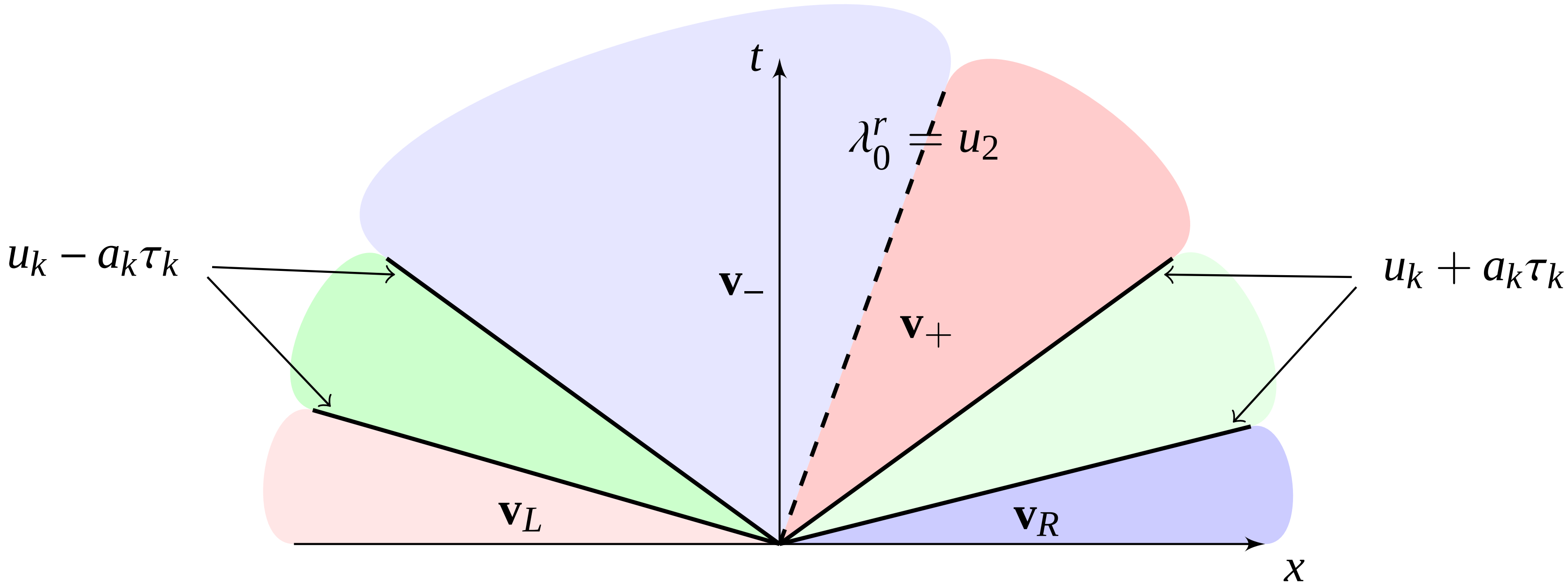
Properties of the Relaxation Model

Five **real** eigenvalues

$$\lambda_1^r = u_1 - a_1\tau_1, \quad \lambda_3^r = u_2 - a_2\tau_2, \qquad \lambda_0^r = u_2, \qquad \lambda_2^r = u_1 + a_1\tau_1, \quad \lambda_4^r = u_2 + a_2\tau_2.$$

Hyperbolic if $u_2 \neq \lambda_k$, $k = 1, 2$.

All the characteristic fields are LD



New Relaxation model

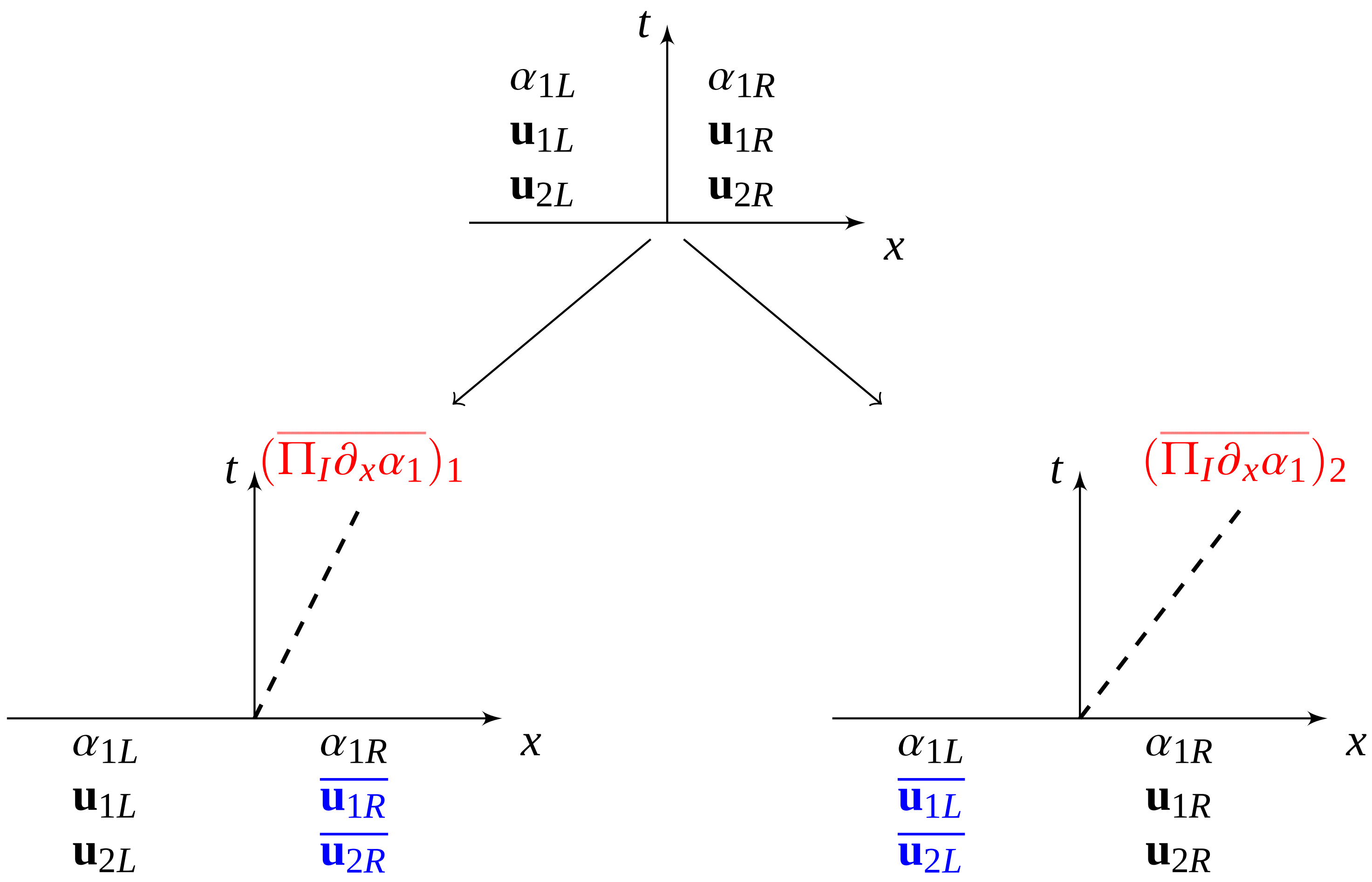
Due to the presence of nonconservative products, the solution of this system is still hard. We propose to consider a modified problem:

$$\begin{cases} \partial_t \alpha_1 + u_2 \partial_x \alpha_1 = 0 \\ \partial_t \alpha_1 \rho_1 + \partial_x \alpha_1 \rho_1 u_1 = 0 \\ \partial_t (\alpha_1 \rho_1 u_1) + \partial_x (\alpha_1 \rho_1 u_1^2 + \alpha_1 \Pi_1) = \overline{\Pi_I \partial_x \alpha_1} \delta_{x - u_2^* t} \\ \partial_t \alpha_2 \rho_2 + \partial_x \alpha_2 \rho_2 u_2 = 0 \\ \partial_t (\alpha_2 \rho_2 u_2) + \partial_x (\alpha_2 \rho_2 u_2^2 + \alpha_2 \Pi_2) = -\overline{\Pi_I \partial_x \alpha_1} \delta_{x - u_2^* t} \\ \partial_t \mathcal{T}_1 + u_2 \partial_x \mathcal{T}_1 = 0 \\ \partial_t \mathcal{T}_2 + u_2 \partial_x \mathcal{T}_2 = 0 \end{cases} \quad (1)$$

The Riemann problem solution is explicitly known

How do we guess $\overline{\Pi_I \partial_x \alpha_1}$?

We chose to be exact on contact discontinuity solutions for the equilibrium system



We need to chose one of the two estimates. *Do we?*

Theorem

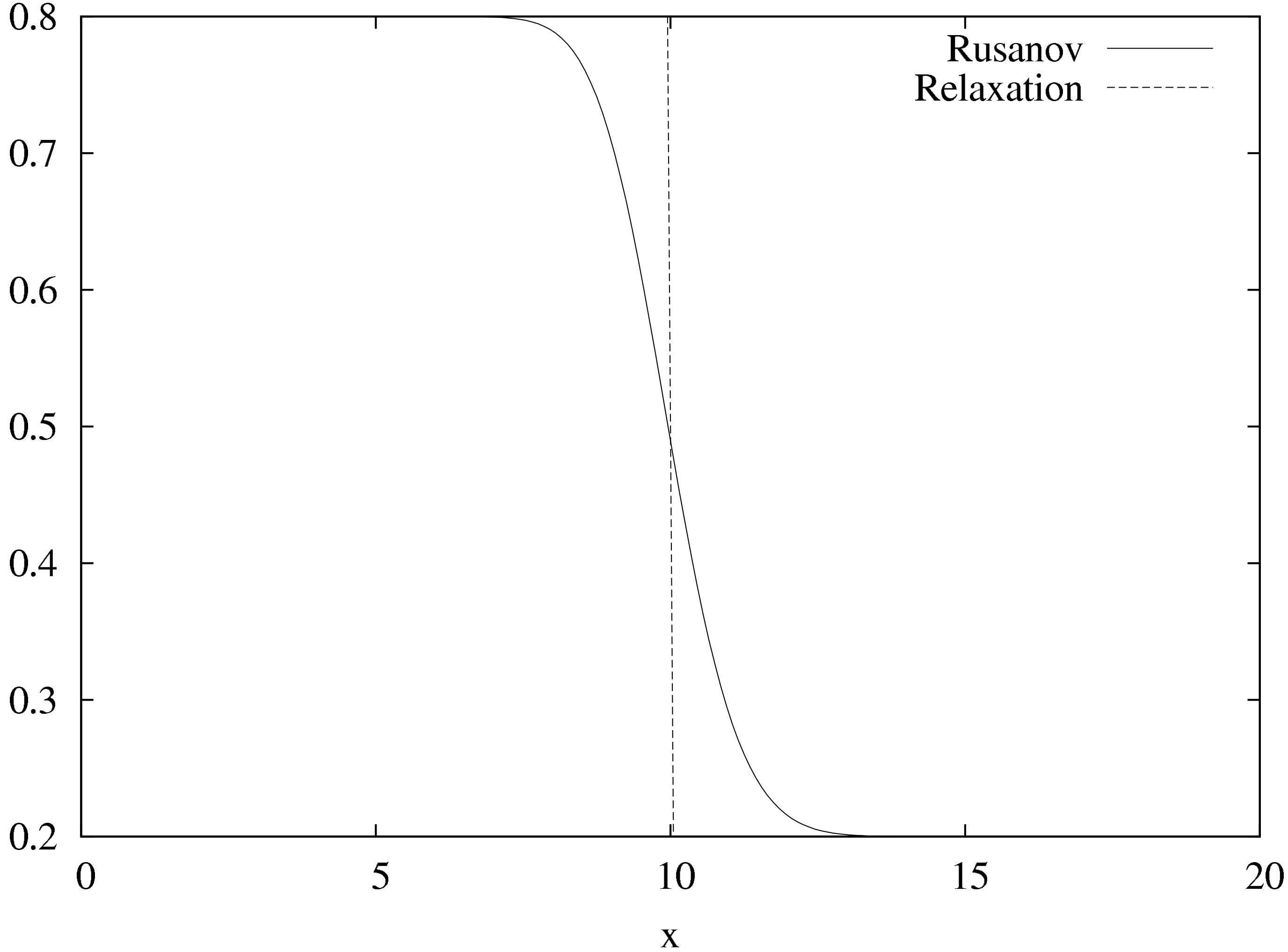
With this choice of $\overline{\Pi_I \partial_x \alpha_1}$, the proposed relaxation method:

(i) (Conservativity): is always conservative on $\alpha_k \rho_k$, $k = 1, 2$ and $\alpha_1 \rho_1 u_1 + \alpha_2 \rho_2 u_2$.

(ii) (L^1 stability): provides numerical solutions that remain in the phase space Ω provided that the free parameters a_k, $k = 1, 2$ are chosen sufficiently large.

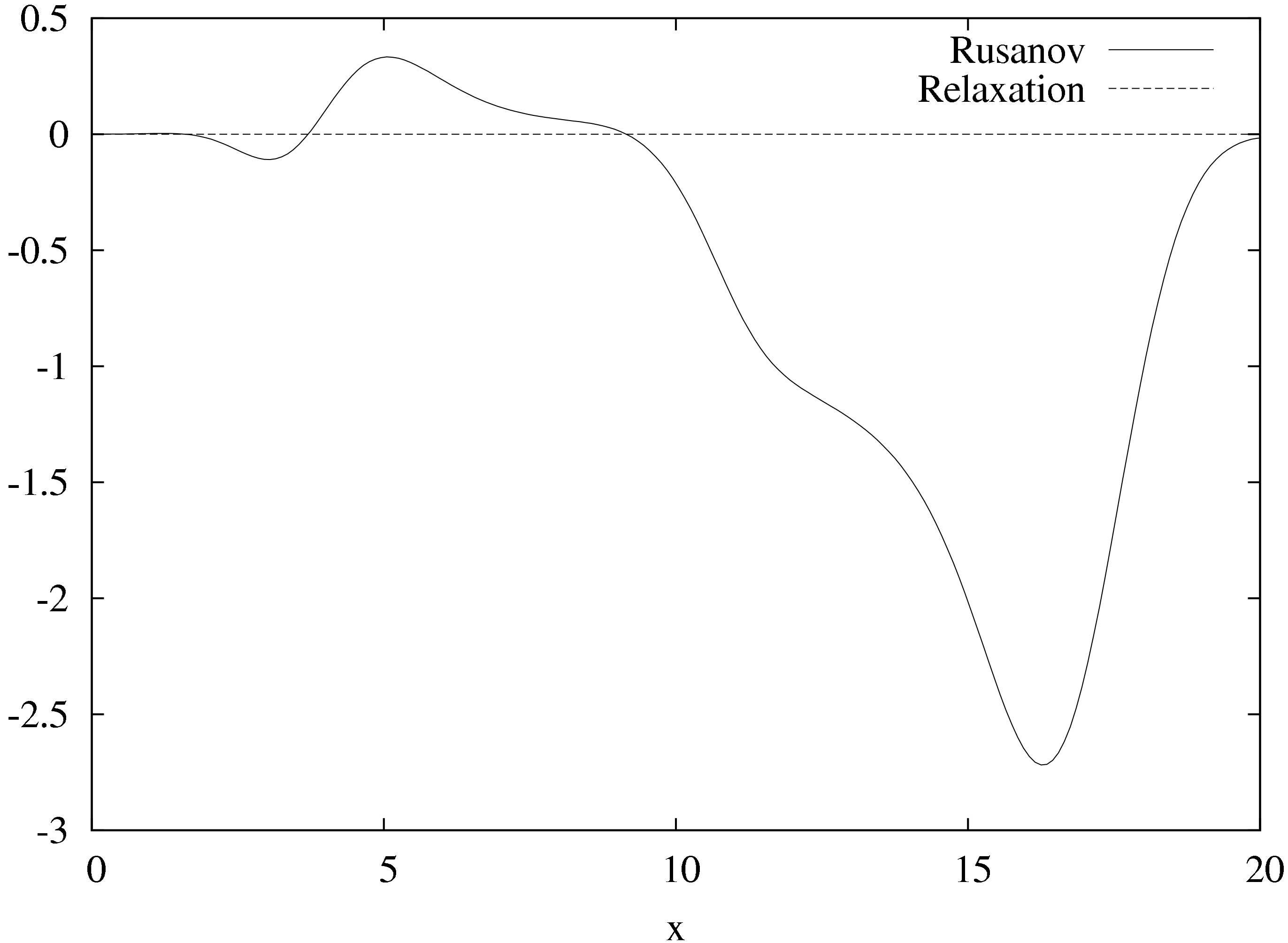
(iii) (Isolated λ_0-contact discontinuities): captures exactly the stationary admissible λ_0-contact discontinuities of the equilibrium system.

Pure stationary contact discontinuity



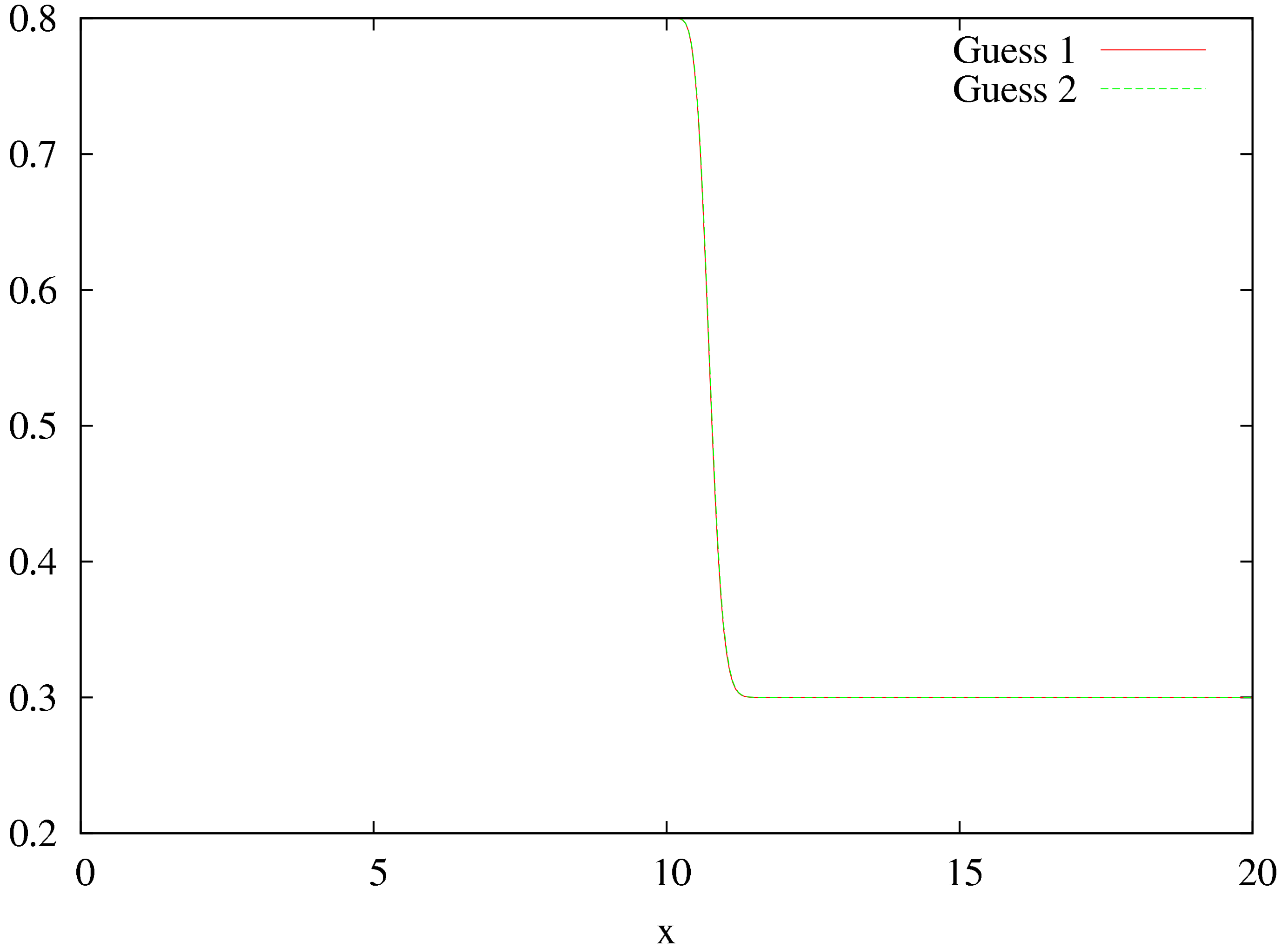
Fraction α_1

Pure stationary contact discontinuity



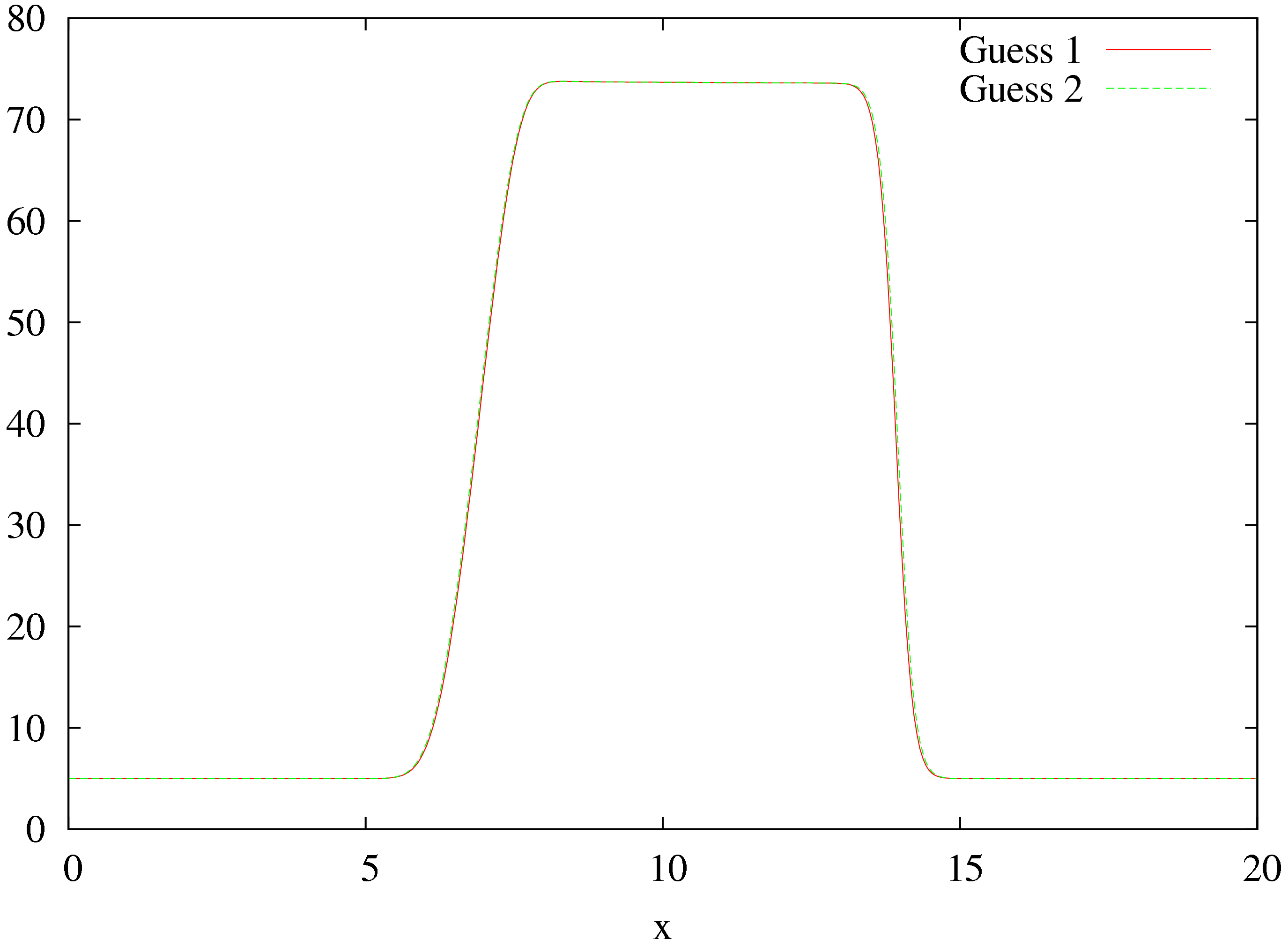
Velocity u_2

Shock Tube - Different Guess



Fraction α_1

Shock Tube - Different Guess



Velocity u_2

Conclusion - Current and Future Work

We presented a relaxation strategy for easily dealing with both the nonlinearities associated with the pressure laws and the nonconservative terms of the two-fluid two-pressure model.
The proposed approximate Riemann solver is given by explicit formulas, preserves the natural phase space, and exactly captures the coupling waves between the two phases.

What's next?

- Treatment of source terms (gravity and friction, in particular) $\longrightarrow$ N. Seguin talk on Thursday afternoon.
- Full two-fluid two-pressure model (with partial energy balance equations) $\longrightarrow$ work in progress.
- Coupling of the two-fluid model with drift models for multiphase flows (multiscale coupling, cf E. Godlewski talk on Thursday morning + J.-M. Hérard, O. Hurisse poster in Session 2).