Benchmark on anisotropic problems
A Galerkin Finite Element Solution

IRSN

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Description of the scheme

All tests take the form of an anisotropic diffusion problem:

$$\begin{cases} -\nabla.(\boldsymbol{K}\nabla u) & \text{in } \Omega, \\ u = \bar{u} & \text{on } \Gamma_D, \\ \boldsymbol{K}\nabla u.\boldsymbol{n} = g & \text{on } \Gamma_N, \end{cases} \tag{1}$$

In a weak sense:

Find $u \in \mathcal{V}$ such that, $\forall v \in \mathcal{V}$,

$$\int_\Omega (\boldsymbol{K}\nabla u).\nabla v = \int_\Omega fv - \int_\Omega (\boldsymbol{K}\nabla \bar{\bar{u}}).\nabla v + \int_{\Gamma_N} gv \tag{2}$$

where $\mathcal{V}$ is the subspace of $H^1(\Omega)$ of functions whose trace vanishes on Γ_D, $\bar{u}$ being appropriately lifted to $\bar{\bar{u}}$.

To solve (1), we use a Galerkin technique :

- replace $\mathcal{V}$ by a finite dimensional $\mathcal{V}_h$,
- replace $\bar{\bar{u}} \in \mathcal{V}$ by the approximate $\bar{\bar{u}}_h \in \mathcal{V}_h$.

On conforming meshes we use standard Lagrange finite elements:

$$\mathcal{V}_h = \left\{ v_h \in \mathcal{V};\ \forall K \in \mathcal{T}_h \left| \begin{array}{l} (v_h)_{|K} \in P^k(K) \text{ on triangle } K \in \mathcal{T}_h \\ (v_h)_{|K} \in Q^k(K) \text{ on quadrilateral } K \in \mathcal{T}_h \end{array} \right. \right\}$$

with $k = 1$ or $k = 2$, where $\mathcal{T}_h$ is a meshing of the domain Ω.

Results for Test 1.1

umin= 0.0, umax=1.0.

- Triangular mesh mesh1

P1 $\rightsquigarrow$ **ocvl2**= 2.00, **ocvgradl2**= 1.00

i	nunkw	nnmat	sumflux	erl2	ergrad	rl2	rgrad
1	21	109	-1.81E+00	3.44E-02	5.88E-01		
2	97	593	-1.12E+00	8.60E-03	2.95E-01	1.81	0.902
3	417	2737	-6.19E-01	2.14E-03	1.47E-01	1.91	0.955
4	1729	11729	-3.25E-01	5.34E-04	7.35E-02	1.95	0.975
5	7041	48529	-1.66E-01	1.33E-04	3.67E-02	1.98	0.989
6	28417	197393	-8.41E-02	3.33E-05	1.84E-02	1.99	0.990
7	114177	796177	-4.23E-02	8.32E-06	9.18E-03	1.99	1.00

i	erflx0	erflx1	erfly0	erfly1	umin	umax
1	1.13E-01	1.13E-01	1.13E-01	1.13E-01	0.00	1.03
2	7.01E-02	7.01E-02	7.01E-02	7.01E-02	0.00	1.01
3	3.87E-02	3.87E-02	3.87E-02	3.87E-02	0.00	1.00
4	2.03E-02	2.03E-02	2.03E-02	2.03E-02	0.00	1.00
5	1.04E-02	1.04E-02	1.04E-02	1.04E-02	0.00	1.00
6	5.26E-03	5.26E-03	5.26E-03	5.26E-03	0.00	1.00
7	2.64E-03	2.64E-03	2.64E-03	2.64E-03	0.00	1.00

P2 $\rightsquigarrow$ **ocvl2**= 3.01, **ocvgradl2**= 2.00

i	nunkw	nnmat	sumflux	erl2	ergrad	rl2	rgrad
1	97	905	1.19E-02	1.54E-03	6.28E-02		
2	417	4337	-6.58E-03	1.80E-04	1.50E-02	2.94	1.96
3	1729	18929	-2.79E-03	2.22E-05	3.69E-03	2.94	1.97
4	7041	79025	-8.37E-04	2.76E-06	9.20E-04	2.97	1.98
5	28417	322865	-2.27E-04	3.45E-07	2.30E-04	2.98	1.99
6	114177	1305137	-5.88E-05	4.31E-08	5.74E-05	2.99	2.00
7	457729	5248049	-1.50E-05	5.38E-09	1.44E-05	3.00	1.99

i	erflx0	erflx1	erfly0	erfly1	umin	umax
1	7.46E-04	7.46E-04	7.46E-04	7.46E-04	0.00	1.00
2	4.11E-04	4.11E-04	4.11E-04	4.11E-04	0.00	1.00
3	1.74E-04	1.74E-04	1.74E-04	1.74E-04	0.00	1.00
4	5.23E-05	5.23E-05	5.23E-05	5.23E-05	0.00	1.00
5	1.42E-05	1.42E-05	1.42E-05	1.42E-05	0.00	1.00
6	3.67E-06	3.68E-06	3.67E-06	3.68E-06	0.00	1.00
7	9.36E-07	9.36E-07	9.36E-07	9.36E-07	0.00	1.00

- Distorted quadrangular mesh mesh4_j_i

Q1

grid	nunkw	nnmat	sumflux	erl2	ergrad
C	256	2116	-1.13E+00	6.71E-02	7.60E-01
F	1024	8836	-4.95E-01	2.50E-02	4.41E-01

grid	erflx0	erflx1	erfly0	erfly1	umin	umax
C	1.02E-01	1.28E-01	4.60E-02	7.21E-03	0.00	0.861
F	4.47E-01	5.23E-01	2.04E-02	6.36E-03	0.00	0.937

Q2

grid	nunkw	nnmat	sumflux	erl2	ergrad
C	1089	16129	8.85E-03	3.51E-04	2.31E-02
F	4225	65025	5.69E-04	4.05E-05	6.35E-03

grid	erflx0	erflx1	erfly0	erfly1	umin	umax
C	6.24E-05	2.31E-03	1.51E-05	5.06E-05	0.00	0.999
F	4.62E-06	1.49E-04	8.26E-07	3.08E-06	0.00	1.00

- **Comments**

Computations on mesh1 show results in accordance with theoretical expectations : convergence orders $\mathcal{O}(h^2)$ in L^2 norm and $\mathcal{O}(h)$ in H^1 norm are attained.
The maximum principle is not preserved on the coarsest grids.

Results for Test 1.2

umin= 0.0, umax=$1 + \sin(1)$.

- Triangular mesh mesh1

P1 $\rightsquigarrow$ **ocvl2**= 2.01, **ocvgradl2**= 1.00

i	nunkw	nnmat	sumflux	erl2	ergrad	rl2	rgrad
1	21	109	1.86E-01	9.35E-03	1.89E-01		
2	97	593	7.56E-02	2.28E-03	9.34E-02	1.84	0.920
3	417	2737	3.30E-02	5.67E-04	4.66E-02	1.91	0.953
4	1729	11729	1.52E-02	1.42E-04	2.33E-02	1.95	0.975
5	7041	48529	7.29E-03	3.54E-05	1.16E-02	1.98	0.993
6	28417	197393	3.56E-03	8.85E-06	5.82E-03	1.99	0.989
7	114177	796177	1.76E-03	2.21E-06	2.91E-03	2.00	0.997

i	erflx0	erflx1	erfly0	erfly1	umin	umax
1	5.51E-02	2.63E-02	2.81E-02	8.64E-02	0.00	1.84
2	2.61E-02	5.64E-03	1.73E-02	4.27E-02	0.00	1.84
3	1.26E-02	8.43E-04	9.38E-03	2.10E-02	0.00	1.84
4	6.16E-03	8.47E-05	4.85E-03	1.03E-02	0.00	1.84
5	3.05E-03	1.71E-04	2.46E-03	5.10E-03	0.00	1.84
6	1.51E-03	1.18E-04	1.24E-03	2.53E-03	0.00	1.84
7	7.55E-04	6.69E-05	6.21E-04	1.26E-03	0.00	1.84

P2 $\rightsquigarrow$ **ocvl2**= 3.01, **ocvgradl2**= 2.00

i	nunkw	nnmat	sumflux	erl2	ergrad	rl2	rgrad
1	97	905	1.14E-03	2.21E-04	9.15E-03		
2	417	4337	-4.90E-04	2.68E-05	2.24E-03	2.89	1.93
3	1729	18929	-2.18E-04	3.32E-06	5.57E-04	2.94	1.96
4	7041	79025	-6.61E-05	4.14E-07	1.39E-04	2.97	1.98
5	28417	322865	-1.80E-05	5.17E-08	3.47E-05	2.98	1.99
6	114177	1305137	-4.68E-06	6.47E-09	8.68E-06	2.99	1.99
7	457729	5248049	-1.19E-06	8.08E-10	2.17E-06	3.00	2.00

i	erflx0	erflx1	erfly0	erfly1	umin	umax
1	1.95E-03	7.66E-03	8.45E-04	3.91E-03	0.00	1.84
2	3.67E-04	1.98E-03	2.82E-04	7.09E-04	0.00	1.84
3	7.60E-05	5.03E-04	7.85E-05	1.44E-04	0.00	1.84
4	1.70E-05	1.26E-04	2.06E-05	3.20E-05	0.00	1.84
5	4.00E-06	3.17E-05	5.26E-06	7.48E-06	0.00	1.84
6	9.68E-07	7.94E-06	1.33E-06	1.81E-06	0.00	1.84
7	2.38E-07	1.99E-06	3.34E-07	4.44E-07	0.00	1.84

- Locally refined mesh mesh3

Q1 $\rightsquigarrow$ **ocvl2**= 2.00, **ocvgradl2**= 1.00

i	nunkw	nnmat	sumflux	erl2	ergrad	rl2	rgrad
1	25	207	2.72E-01	5.06E-03	6.51E-02		
2	129	1547	1.32E-01	1.26E-03	3.20E-02	1.69	0.867
3	577	8035	6.47E-02	3.14E-04	1.58E-02	1.86	0.941
4	2433	36179	3.20E-02	7.84E-05	7.90E-03	1.93	0.963
5	9985	153139	1.59E-02	1.96E-05	3.94E-03	1.96	0.985

i	erflx0	erflx1	erfly0	erfly1	umin	umax
1	4.26E-02	4.36E-02	4.84E-03	1.35E-01	-3.50E-02	1.84
2	2.24E-02	1.14E-02	3.24E-03	6.54E-02	-1.03E-02	1.84
3	1.15E-02	2.91E-03	1.84E-03	3.20E-02	-2.81E-03	1.84
4	5.83E-03	7.35E-04	9.73E-04	1.58E-02	-7.37E-04	1.84
5	2.93E-03	1.85E-04	5.00E-04	7.87E-03	-1.88E-04	1.84

Q2 $\rightsquigarrow$ **ocvl2**= 3.00, **ocvgradl2**= 2.00

i	nunkw	nnmat	sumflux	erl2	ergrad	rl2	rgrad
1	129	2257	-7.24E-03	1.31E-04	4.12E-03		
2	577	12865	-2.01E-03	1.62E-05	1.01E-03	2.79	1.88
3	2433	60449	-5.20E-04	2.02E-06	2.52E-04	2.89	1.93
4	9985	260833	-1.32E-04	2.52E-07	6.28E-05	2.95	1.97
5	40449	1082465	-3.32E-05	3.14E-08	1.57E-05	2.98	1.98

i	erflx0	erflx1	erfly0	erfly1	umin	umax
1	1.31E-03	1.74E-02	9.52E-05	2.84E-03	-6.56E-04	1.84
2	3.04E-04	4.46E-03	2.84E-05	6.55E-04	-8.26E-05	1.84
3	7.36E-05	1.13E-03	7.37E-06	1.62E-04	-1.05E-05	1.84
4	1.81E-05	2.83E-04	1.85E-06	4.05E-05	-1.37E-06	1.84
5	4.49E-06	7.10E-05	4.61E-07	1.01E-05	-1.75E-07	1.84

- **Comments**

Convergence orders remain as expected on the conforming and non-conforming meshes, but the maximum principle is only preserved on the conforming ones, where solution extrema are exactly determined.

Results for Test 2 Numerical locking

Triangular mesh mesh1. umin= -1, umax= 1.

- $\delta = 10^5$

P1 $\rightsquigarrow$ **ocvl2**= 2.00, **ocvgradl2**= 1.01

i	nunkw	nnmat	sumflux	erl2	ergrad	rl2	rgrad
1	37	221	3.22E+00	7.00E-01	4.39E+00		
2	129	833	9.37E-01	1.49E-01	1.91E+00	2.48	1.33
3	481	3233	5.19E-02	3.89E-02	9.87E-01	2.04	1.01
4	1857	12737	-1.73E-03	9.82E-03	4.97E-01	2.04	1.02

i	erflx0	erflx1	fly0	fly1	umin	umax
1	1.00E-00	1.00E-00	1.37E+00	1.85E+00	-9.48E-03	9.75E-03
2	3.64E-01	3.63E-01	4.18E-01	5.25E-01	-1.00	1.00
3	1.00E-01	9.93E-02	2.08E-02	3.54E-02	-1.00	1.00
4	2.57E-02	2.53E-02	-6.20E-04	1.25E-03	-1.00	1.00

P2 $\rightsquigarrow$ **ocvl2**= 2.97, **ocvgradl2**= 2.00

i	nunkw	nnmat	sumflux	erl2	ergrad	rl2	rgrad
1	129	1353	-1.69E-01	2.79E-02	5.29E-01		
2	481	5281	-1.14E-01	1.49E-02	3.88E-01	0.95	0.47
3	1857	20865	-3.02E-02	1.92E-03	9.96E-02	3.03	2.01
4	7297	82945	-4.07E-03	2.42E-04	2.51E-02	3.03	2.01

i	erflx0	erflx1	fly0	fly1	umin	umax
1	2.15E-01	2.17E-01	-5.59E-02	-1.05E-01	-0.956	0.956
2	1.58E-01	1.58E-01	-4.16E-02	-7.07E-02	-1.00	1.00
3	4.80E-02	4.81E-02	-1.27E-02	-1.72E-02	-1.00	1.00
4	1.26E-02	1.26E-02	-1.72E-03	-2.32E-03	-1.00	1.00

- $\delta = 10^6$

P1 $\rightsquigarrow$ **ocvl2**= 2.00, **ocvgradl2**= 0.995

i	nunkw	nnmat	sumflux	erl2	ergrad	rl2	rgrad
1	37	221	1.03E+01	7.05E-01	4.42E+00		
2	129	833	3.01E+00	1.50E-01	1.92E+00	2.48	1.33
3	481	3233	1.80E-01	3.92E-02	9.94E-01	2.04	1.00
4	1857	12737	1.86E-03	9.89E-03	5.01E-01	2.04	1.01

i	erflx0	erflx1	fly0	fly1	umin	umax
1	1.00E-00	1.00E+00	4.91E+00	5.39E+00	-2.93E-03	3.02E-03
2	3.64E-01	3.63E-01	1.45E+00	1.56E+00	-1.00	1.00
3	9.98E-02	9.96E-02	8.33E-02	9.80E-02	-1.00	1.00
4	2.56E-02	2.54E-02	3.64E-04	2.24E-03	-1.00	1.00

P2 $\rightsquigarrow$ **ocvl2**= 2.96, **ocvgradl2**= 2.00

i	nunkw	nnmat	sumflux	erl2	ergrad	rl2	rgrad
1	129	1353	-5.16E-01	2.81E-02	5.33E-01		
2	481	5281	-3.59E-01	1.51E-02	3.93E-01	0.94	0.46
3	1857	20865	-9.59E-02	1.94E-03	1.01E-01	3.04	2.01
4	7297	82945	-1.31E-02	2.45E-04	2.54E-02	3.02	2.02

i	erflx0	erflx1	fly0	fly1	umin	umax
1	2.16E-01	2.16E-01	-2.32E-01	-2.81E-01	-0.95	0.95
2	1.58E-01	1.58E-01	-1.64E-01	-1.94E-01	-1.00	1.00
3	4.83E-02	4.83E-02	-4.56E-02	-5.02E-02	-1.00	1.00
4	1.26E-02	1.26E-02	-6.22E-03	-6.83E-03	-1.00	1.00

- **Comments**

Although theoretical convergence orders and the maximum principle are preserved for both values of the permeability coefficient δ, numerical locking is observed in both cases on the coarsest mesh.

Results for Test 3 : Oblique flow

- Uniform rectangular mesh mesh2. umin= 0.0, umax= 1.0.

Q1

i	nunkw	nnmat	sumflux	umin	umax
1	9	49	-7.29E-16	0.00	1.00
2	49	361	-1.25E-15	0.00	1.00
3	225	1849	3.79E-16	0.00	1.00
4	961	8281	2.37E-15	0.00	1.00
5	3969	34969	3.55E-14	0.00	1.00
ref	249001	2235025	-2.18E-13	0.00	1.00

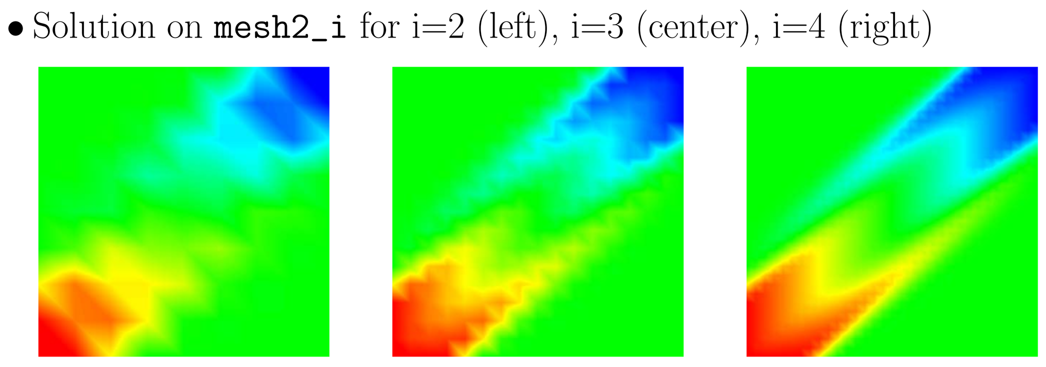
i	flux0	flux1	fluy0	fluy1	ener1	ener2	eren
1	-0.305	0.305	-0.250	0.250	0.221	0.349	0.367
2	-0.196	0.196	-0.125	0.125	0.267	0.255	0.0449
3	-0.198	0.198	-0.111	0.111	0.300	0.310	0.0323
4	-0.197	0.197	-0.104	0.104	0.270	0.300	0.100
5	-0.196	0.196	-0.101	0.101	0.255	0.280	0.0893
ref	-0.193	0.193	-0.0990	0.0990	0.243	0.247	0.0162

Q2

i	nunkw	nnmat	sumflux	umin	umax
1	49	529	-4.51E-16	0.00	1.00
2	225	3025	-1.86E-15	0.00	1.00
3	961	14161	-2.52E-15	0.00	1.00
4	3969	61009	-1.90E-14	0.00	1.00
5	16129	253009	-7.52E-14	0.00	1.00
ref	998001	15928081	-4.45E-12	0.00	1.00

i	flux0	flux1	fluy0	fluy1	ener1	ener2	eren
1	-0.127	0.127	-0.0360	0.0360	0.264	0.174	0.341
2	-0.193	0.193	-0.0940	0.0940	0.281	0.319	0.119
3	-0.192	0.192	-0.0961	0.0961	0.248	0.260	0.0462
4	-0.192	0.192	-0.0975	0.0975	0.245	0.248	0.0121
5	-0.193	0.193	-0.0981	0.0981	0.243	0.243	0.00
ref	-0.193	0.193	-0.0986	0.0986	0.242	0.242	0.00

- Solution on mesh2_i for i=2 (left), i=3 (center), i=4 (right)



- **Comments**

The aim of this case was to test the ability of the schemes to verify the maximum principle. The finite element method behaves very well in this respect and is exact on solution extrema.

Results for Test 4 : Vertical fault

- Non conforming rectangular mesh mesh5. umin= 0.0, umax= 1.0.

Q1

i	nunkw	nnmat	sumflux	umin	umax
reg	361	3025	2.52E+00	0.00	1.00
ref	101761	912025	1.56E-01	0.00	1.00

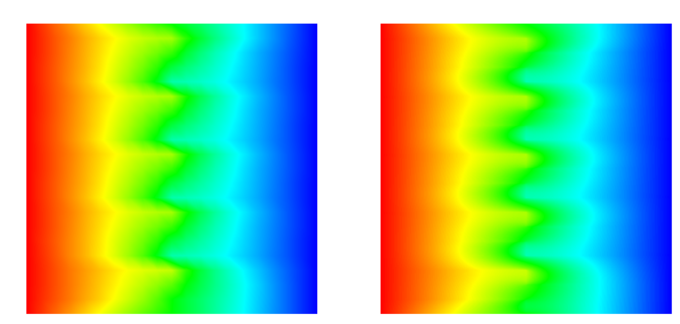
i	flux0	flux1	fluy0	fluy1	ener1	ener2	eren
reg	-43.3	45.4	0.433	8.26E-04	44.5	43.2	2.92E-02
ref	-42.2	44.5	-2.16	7.96E-04	43.3	43.2	2.31E-03

Q2

i	nunkw	nnmat	sumflux	umin	umax
reg	1521	22801	-7.07E-03	0.00	1.00
ref	408321	6507601	-2.60E-05	0.00	1.00

i	flux0	flux1	fluy0	fluy1	ener1	ener2	eren
reg	-42.4	44.7	-2.28	7.84E-04	43.5	43.4	2.30E-03
ref	-42.1	44.5	-2.32	7.96E-04	43.2	43.2	0.00E+00

- Solution for the vertical fault on the meshes: (Left) mesh5_reg. (Right) mesh5_ref.



- **Comments**

For this test, we only conducted calculations on conforming meshes. The energy computed by the Q2 finite element scheme on the reference mesh is the exact value of the continuous energy, 43.2.

Results for Test 5 : Heterogeneous rotating anisotropy

- Non conforming rectangular mesh mesh5. umin= 0.0, umax= 1.0.

Q1 $\rightsquigarrow$ **ocvl2**= 2.01, **ocvgradl2**= 1.00

i	nunkw	nnmat	sumflux	erl2	ergrad	rl2	rgrad
1	9	49	-5.06E-01	3.07E-02	5.04E-01		
2	49	361	-1.28E-01	7.89E-03	2.52E-01	1.60	0.819
3	225	1849	-3.20E-02	2.00E-03	1.26E-01	1.80	0.909
4	961	8281	-7.97E-03	5.04E-04	6.30E-02	1.90	0.955
5	3969	34969	-1.99E-03	1.26E-04	3.15E-02	1.95	0.977

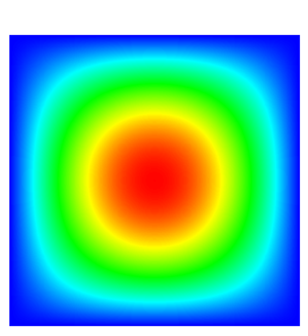
i	erflx0	erflx1	erfly0	erfly1	umin	umax
1	1.04E-01	8.46E-02	1.04E-01	8.46E-02	0.00	1.05
2	2.73E-02	1.64E-02	2.73E-02	1.64E-02	0.00	1.01
3	6.97E-03	3.36E-03	6.97E-03	3.36E-03	0.00	1.00
4	1.76E-03	7.46E-04	1.76E-03	7.46E-04	0.00	1.00
5	4.41E-04	1.75E-04	4.41E-04	1.75E-04	0.00	1.00

Q2 $\rightsquigarrow$ **ocvl2**= 3.02, **ocvgradl2**= 2.02

i	nunkw	nnmat	sumflux	erl2	ergrad	rl2	rgrad
1	49	529	2.98E-01	4.45E-03	1.04E-01		
2	225	3025	7.89E-02	5.58E-04	2.65E-02	2.72	1.79
3	961	14161	2.00E-02	7.00E-05	6.63E-03	2.86	1.91
4	3969	61009	5.03E-03	8.78E-06	1.66E-03	2.93	1.95
5	16129	253009	1.26E-03	1.10E-06	4.15E-04	2.96	1.98

i	erflx0	erflx1	erfly0	erfly1	umin	umax
1	6.70E-02	5.59E-02	6.89E-02	5.51E-02	0.00	1.00
2	1.80E-02	1.35E-02	1.83E-02	1.35E-02	0.00	1.00
3	4.60E-03	3.30E-03	4.68E-03	3.30E-03	0.00	1.00
4	1.16E-03	8.15E-04	1.18E-03	8.15E-04	0.00	1.00
5	2.90E-04	2.02E-04	2.95E-04	2.02E-04	0.00	1.00

- Solution of Test 5



- **Comments**

Computations show results in accordance with theoretical expectations : convergence orders $\mathcal{O}(h^2)$ in L^2 norm and $\mathcal{O}(h)$ in H^1 norm are attained.
The maximum principle is not preserved on the coarsest grids.

Results for Test 6 and Test 7

- **Test 6**

Oblique drain, min $= -1.2$, max $= 0$, coarse oblique mesh mesh6

Q1

nunkw	nnmat	sumflux	erl2	ergrad
180	1450	3.31E-14	3.28E-16	6.52E-15

erflx0	erflx1	erfly0	erfly1	umin	umax
1.34E-15	2.99E-15	1.39E-15	1.53E-15	-1.20E+00	0.00E+00

Q2

nunkw	nnmat	sumflux	erl2	ergrad
779	11289	-9.65E-14	4.54E-16	1.12E-14

erflx0	erflx1	erfly0	erfly1	umin	umax
1.19E-15	1.73E-14	1.39E-16	7.22E-15	-1.20E+00	0.00E+00

- **Test 7**

Oblique barrier, min $= -5.575$, max $= 0.575$, coarse oblique mesh mesh6

Q1

nunkw	nnmat	sumflux	erl2	ergrad
180	1450	-6.48E-14	3.79E-15	6.49E-14

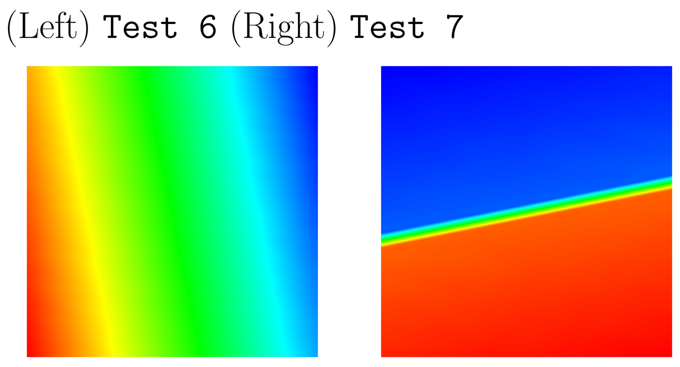
erflx0	erflx1	erfly0	erfly1	umin	umax
5.84E-14	7.98E-14	4.44E-16	3.62E-14	-5.575	0.575

Q2

nunkw	nnmat	sumflux	erl2	ergrad
779	11289	5.75E-15	6.12E-15	8.19E-14

erflx0	erflx1	erfly0	erfly1	umin	umax
5.59E-14	7.85E-14	2.00E-15	3.00E-14	-5.575	0.575

- Solutions: (Left) Test 6 (Right) Test 7



- **Comments**

For these tests, the solution is linear, and thus belongs to the approximation space. Hence, the finite element method is exact.

Results for Test 8 and Test 9

- **Test 8**

Perturbed parallelograms mesh mesh8, umin= 0.0, umax= 1.0

Q1

nunkw	nnmat	sumflux	umin	umax
100	784	3.53E-02	-4.17E-03	4.90E-02

flux0	flux1	fluy0	fluy1
5.51E-06	7.15E-05	5.46E-01	4.89E-01

Q2

nunkw	nnmat	sumflux	umin	umax
441	6241	-8.43E-04	-5.07E-03	8.04E-02

flux0	flux1	fluy0	fluy1
-5.52E-05	1.96E-05	4.98E-01	5.01E-01

- **Test 9**

Anisotropy with wells. Square uniform grid mesh9. umin= 0.0, umax= 1.0

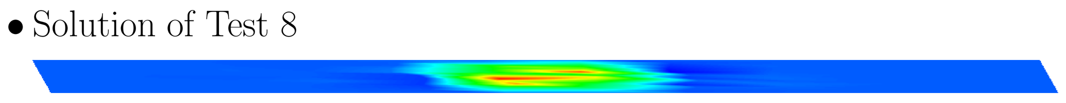
Q1

nunkw	nnmat	sumflux	umin	umax
136	1044	-2.64E-10	-2.36E-02	1.02E+00

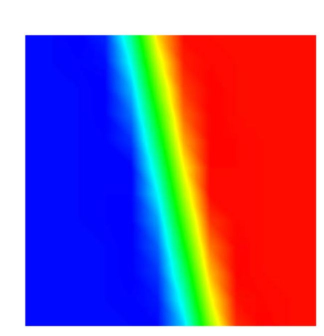
Q2

nunkw	nnmat	sumflux	umin	umax
511	7407	-2.72E-11	-5.94E-03	1.01E+00

- Solution of Test 8



- Solution of Test 9



- **Comments**

In Test 8, the approximate solution shows internal oscillations. In both tests, the maximum principle is violated.