Numerical Modelling of Compressible Flows Using Pressure-Correction Algorithms

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Compressible flows with shock solutions are encountered in a wide variety of situations, from aeronautics to power generation industry. It is well known that Godunov-type methods provide very good approximations for such flows ([6], [7] for example). However, some existing industrial software that are based on pressure correction methods may have to deal with such configurations. Two pressure correction schemes are considered ([1] and [10], [8]). We show numerically on several academic test-cases that they can capture the exact shock solutions of Euler equations provided conservativity in time and space is ensured (see also [5] and [9]). We finally compare the pressure correction schemes to the approximate Godunov scheme VFRoe-ncv [2] on a test-case with a strong local and constant heat source term. This test-case, for which no analytical solution is available, represents the Joule effect due to an unwanted electric arc in an electrical power transformer (see [3]). If conservativity is ensured, all schemes converge towards the same solution as the mesh is refined.

System of equations

We consider the Euler equations with perfect gas thermodynamics under form (1) or (2):

$$(1)\left\{\begin{array}{lcl} \partial_t \rho + \partial_x(\rho\, u) & = & 0 \\ \partial_t(\rho\, u) + \partial_x(\rho\, u\, u) + \partial_x P & = & 0 \\ \partial_t(\rho\, H) + \partial_x(\rho\, u\, H) & = & \partial_t P + \Phi \end{array}\right. \qquad (2)\left\{\begin{array}{lcl} \partial_t \rho + \partial_x(\rho\, u) & = & 0 \\ \partial_t(\rho\, u) + \partial_x(\rho\, u\, u) + \partial_x P & = & 0 \\ \partial_t(\rho\, E) + \partial_x(\rho\, u\, E + u\, P) & = & \Phi \end{array}\right.$$

An iterative pressure-correction algorithm: NLK

At each time step, the algorithm solves system (1) in three steps with sub-iterations:

m-cycle (1:ntg)

Velocity *prediction* $\rho^n \dfrac{u_m^* - u^n}{\Delta t} - u_m^*\, \partial_x Q_m^{n+1} + \partial_x(q^n\, u_m^*) = -\partial_x P^n$

k-cycle (1:nthm)

Enthalpy *step* $\rho^n \dfrac{H_{m,k+1}^{n+1} - H^n}{\Delta t} - H_{m,k+1}^{n+1}\, \partial_x q_{m,k}^{n+1} + \partial_x(q_{m,k}^{n+1}\, H_{m,k+1}^{n+1}) = \dfrac{P_{m,k}^{n+1} - P^n}{\Delta t} + \Phi$

Update density

Stopping tests *for k-cycles:* *discrete mass equation is satisfied*
for m-cycles: *discrete divergence of* Q_m^{n+1} *is unchanged*

Pressure *step* $\dfrac{P_{m,k+1}^{n+1} - P_{m,k}^{n+1}}{(c^2)_{m,k+1/2}^{n+1}\Delta t} - \partial_x \Delta t \partial_x(P_{m,k+1}^{n+1} - P_{m,k}^{n+1}) = -\dfrac{\rho_{m,k+1/2}^{n+1} - \rho^n}{\Delta t} - \partial_x q_{m,k}^{n+1}$

Update velocity, mass flux, density

Update mass flow rate for next m iteration $Q_{m+1}^{n+1} = q_{m,k_{max}}^{n+1}$

Conservativity *in time is ensured at convergence of the m- and k-cycles since:*

$$\rho^n \frac{u_{m_{max}}^* - u^n}{\Delta t} - u_{m_{max}}^*\, \partial_x Q_{m_{max}}^{n+1} = \frac{\rho^{n+1}\, u^{n+1} - \rho^n\, u^n}{\Delta t}$$

A non iterative pressure-correction algorithm: SLK

At each time step, the algorithm solves system (2) in three steps without sub-iteration:

Density *step* $\dfrac{\rho^{n+1} - \rho^n}{\Delta t} + \partial_x\left(q^n - \Delta t\, \theta\, \beta\, \partial_x s^n\right) - \partial_x\left(\Delta t(c^2\, \partial_x \rho^{n+1})\right) = 0$

Update the convective mass flux $q_{ac}^{n+1} = q^n - \Delta t\, \left(c^2\, \partial_x \rho^{n+1} + \theta\, \beta\, \partial_x s^n\right)$

Velocity *step* $\rho^n \dfrac{u^{n+1} - u^n}{\Delta t} - u^{n+1}\, \partial_x q_{ac}^{n+1} + \partial_x(q_{ac}^{n+1}\, u^{n+1}) = -\partial_x P^n$

Energy *step* $\rho^n \dfrac{E^{n+1} - E^n}{\Delta t} - E^{n+1}\, \partial_x q_{ac}^{n+1} + \partial_x\left(q_{ac}^{n+1}\left(E^{n+1} + \dfrac{P^n}{\rho^{n+1}}\right)\right) = \Phi$

Update the pressure

Conservativity *in time is ensured thanks to the definition of the convective mass flux.*

Analytical test-cases without heat source term

We consider a contact discontinuity, the Sod shock tube, a double expansion, a double shock and a sonic shock tube. The behaviour of so-called first order schemes is recovered ([4]): NLK operated with a sufficient number of subcycles and SLK both capture the exact solution computed following [11]. We outline that NLK operated with too few subcycles is no more conservative in time and does not converge towards the exact solution for cases involving a shock.

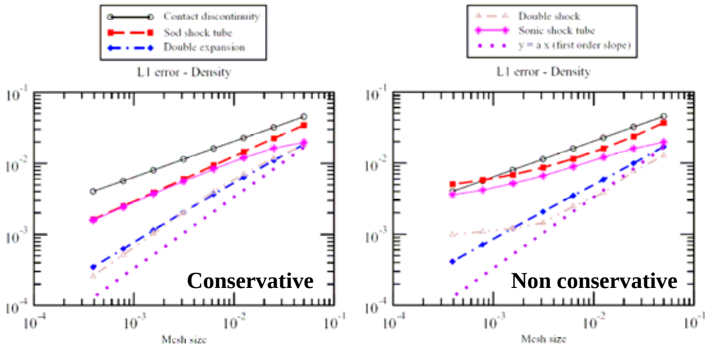
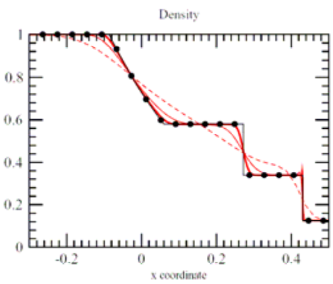


Fig. 1: evolution of the L^1-error for the density on the analytical test-cases without heat source term. Plots show the behaviour for SLK (left) and for NLK with k_{max}=20 and m_{max}=1 (right).



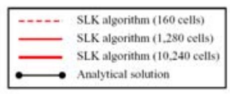


Fig 2: density profile for the sonic shock tube [12] calculated with SLK on different meshes. No spurious phenomenon is observed at the location of the initial discontinuity.

Test-case with a constant heat source term

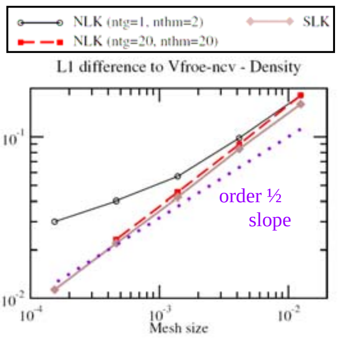


Fig 3: evolution of the L^1-difference to VFRoe-ncv for the density (test-case with a constant heat source term).

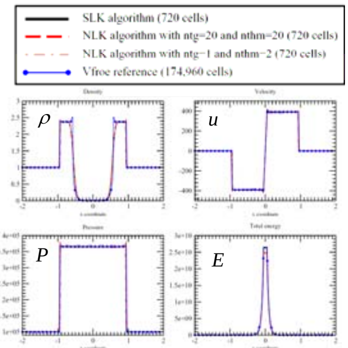


Fig 4: illustration of the profiles of the density, velocity, pressure and energy obtained for the test-case with a constant heat source term.

[1] F. Archambeau, N. Méchitoua, M. Sakiz, *Code_Saturne*: A finite volume code for the computation of turbulent incompressible flows - Industrial applications, *International Journal on Finite Volumes*, http://averoes.math.univ-paris13.fr/, Volume 1, 2004.
[2] T. Buffard, T. Gallouët, J.-M. Hérard, A sequel to a rough Godunov scheme: application to real gases, *Computers and Fluids*, vol. 29, n. 7, pp. 813-847, 2000.
[3] A. Douce, C. Delalondre, H. Biausser, J.B. Guillot, Numerical modelling of an anodic metal bath heated with an Argon transferred arc, *The Int. Iron and Steel Institute of Japan*, vol. 43 (8), pp. 1128-1135, 2003.
[4] T. Gallouët, J.-M. Hérard, N. Seguin, Some recent finite volume schemes to compute Euler equations using real gas EOS, *Int. Journal for Numerical Methods in Fluids*, vol. 39, n. 12, pp. 1073-1138, 2002.
[5] T. Gallouët, J.-M. Hérard, N. Seguin, A hybrid scheme to compute contact discontinuities in one dimensional Euler systems, *Math. Model. and Numerical Analysis*, vol. 36, pp. 1133-1159, 2002.
[6] E. Godlewski, P.-A. Raviart, *Numerical approximation of hyperbolic systems of conservation laws*, New York, Springer-Verlag, 1996.
[7] S.K. Godunov, Finite difference method for numerical computation of discontinuous solutions of the equations of fluid dynamics, *Mat. Sb.*, vol. 47, pp. 271-300, 1959.
[8] A. Guelfi, D. Bestion, M. Boucker, P. Boudier, P. Fillion, M. Grandotto, J.-M. Hérard, E. Hervieu, P. Péturaud, NEPTUNE - A new software platform for advanced nuclear thermal hydraulics, *Nuclear Science and Engineering*, vol. 156, pp. 281-324, 2007.
[9] X. Hou, P.G. Le Floch, Why non conservative schemes converge to wrong solutions: error analysis, *Mathematics of Computation*, vol. 62, pp. 497-530, 1994.
[10] P. Mathon, F. Archambeau, J.-M. Hérard, Implantation d'un algorithme compressible dans *Code_Saturne, EDF internal report HI83-03/016*, in French, unpublished, 2003.
[11] J. Smoller, *Shock waves and reaction-diffusion equations*, New York, Springer-Verlag, 1983.
[12] E. Toro, *Riemann solvers and numerical methods for fluid dynamics*, Berlin, Springer-Verlag, 1999.

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