Flux-based level set method for two-phase flow problem

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Outline

- o Level set methods
 - o application to two-phase flow problem

- o Numerical difficulties
 - o "mass conservation"
- o Flux-based level set method
- o Benchmarks

Two-phase flow

o *interface* between material discontinuities

o density, viscosity

o interface condition (here simplified):

 pressure jump in normal direction is proportional to curvature

o dynamic interface

o continuous nonzero velocity at the interface



Naegele, Wittum: *... methods for the incompressible Navier-Stokes equations,* JCP 2007 **Frolkovič, Logashenko, Wittum**: *... level set method for two-phase flow,* FVCA 2008

Two-phase flow

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Level set function

o the interface is given implicitly using a smooth function

o example of a circular interface:

o choose a level set function:

$$\phi(x, y) := x^2 + y^2 - 0.8^2$$

o indicate points inside
o indicate points outside
o capture interface φ = 0



Level set function

o the interface is given implicitly using a smooth function

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o the interface condition

o normal unit vectors

 $\vec{N} = \frac{\nabla \phi}{|\nabla \phi|}$



Level set function

o the interface is given implicitly using a smooth function

o the interface condition

o normal unit vectors

 $\vec{N} = \frac{\nabla \phi}{|\nabla \phi|}$

o mean curvature

 $\kappa = \nabla \cdot \vec{N}$



Frolkovič, Logashenko, Wittum: ... level set method for two-phase flow, FVCA 2008

Level set equation

o the dynamic interface according to advection equation

$$\mathbf{0} = \partial_t \phi + \vec{V} \cdot \nabla \phi = \partial_t \phi + v_1 \, \partial_x \phi + v_2 \, \partial_y \phi$$

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o the level set function is constant along characteristic curves

$$\frac{dx(t)}{dt} = v_1, \ \frac{dy(t)}{dt} = v_2$$

$$\phi(t, x(t), y(t)) \equiv const$$

Nonlinear level set equation

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• e.g., expanding of the circle: $\partial_t \phi + \frac{\nabla \phi}{|\nabla \phi|} \cdot \nabla \phi = 0$



Nonlinear level set equation

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• e.g., expanding of the circle: $\partial_t \phi + \frac{\nabla \phi}{|\nabla \phi|} \cdot \nabla \phi = 0$



Two-phase flow - reinitialization of the level set function

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o without and with reinitialization:



Two-phase flow - reinitialization of the level set function

o without and with reinitialization:



Finite Volumes for Complex Applications, Aussois, 10.6.2008

Numerical difficulties

Two-phase flow

Sussman, Smereka, Osher: A level set approach for computing solutions to incompressible two-phase flow; 1994 (second order ENO scheme, ...)

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o Matlab Levelset Toolbox [Mitchell 2004]

 $\partial_t \phi + \vec{V} \cdot \nabla \phi = 0$

- o rotation of a circle
- o 2nd order ENO
- o grid 20x20

Applications and numerical difficulties

Two-phase flow

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 - o loss of the area!



Applications and numerical difficulties

Two-phase flow

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 $\partial_t \phi + \vec{V} \cdot \nabla \phi = 0$

- o rotation of a circle
- o 2nd order ENO
- o grid 20x20
 - o loss of the area!

Losasso, Fedkiw, Osher (2006): "Interestingly, this work also demonstrated the largest weakness of the level set method, i.e. mass or information loss characteristic of most Eulerian capturing techniques."

Numerical difficulties

Two-phase flow - improvements

Sussman, Fatemi, Smereka, Osher: *An improved level set method for incompressible two-phase flow*; 1995 (higher order ENO scheme, ...)

Sussman, Fatemi: An efficient interface-preserving level set redistancing algorithm ..., 1999

Sussman, Puckett: A coupled level set and volume of fluid method ..., 2000

Enright, Fedkiw, Ferziger, Mitchell: A hybrid particle level set method ..., 2002

Hieber, Koumoutsakos: A Lagrangian particle level set method, 2005

Losasso, Fedkiw, Osher: Spatially Adaptive Techniques for Level Set Methods and Incompressible Flow, 2006 - review

$$\partial_t \phi + \vec{V} \cdot \nabla \phi = 0$$

Conservative schemes

Frolkovic, Mikula: *Flux-based level set method: a finite volume method for evolving interfaces*, Preprint 2003, Applied Numerical Mathematics 2007 (not for two phase flow)

Olsson, Kreiss: A conservative level set method for two phase flow, JCP 2005 (finite volume method with limiter and artificial compressibility)

Marchandise, Remacle, Chevaugeon: A quadrature-free discontinuous Galerkin method for the level set equation, JCP 2005

Di Pietro, Lo Forte, Parolini: *Mass preserving finite element implementations* of the level set method, ANM 2006

Zheng, Lowengrub, Anderson, Cristini: *Adaptive unstructured volume remeshing...,* JCP 2005 (discontinuous Galerkin)

$$\partial_t \phi + \nabla \cdot \left(\phi \vec{V} \right) = \phi \nabla \cdot \vec{V}$$

Numerical difficulties



Fig. 1. 0.05, 0.5 and 0.95 Contours of Φ initially and after one revolution using different numerical methods. (c)–(f) are second order TVD methods, (b) is second order but not TVD. (a) Initial state; (b) centered differences; (c) upwind with Minmod; (d) upwind with Van Albada; (e) upwind with Van Leer; (f) upwind with Superbee.

Olsson, Kreiss: A conservative level set method for two phase flow, JCP 2005

http://www.math.sk/frolkovi

1D illustration of the idea

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o standard upwind approach:

$$\phi^{n+1}(x_i) = ?$$

1D illustration of the idea



• standard upwind approach (does not require the gradient of solution!): $\phi^{n+1}(x_i) = \phi^n(x_i - \Delta t^n V)$

1D illustration of the idea



o standard upwind approach:

$$\phi^{n+1}(x_i) = \phi^n(x_i - \Delta t^n V)$$

o the level set approach (does require the gradient of solution!):

$$\phi^{n+1}(x_i) = \phi^n(x_i) - \Delta t^n V \partial_x \phi$$

1D illustration of the idea



o standard upwind approach and Taylor expansion

$$\phi^n(x_i - \Delta t^n V) = \phi^n(x_i) - \Delta t^n V \partial_x \phi$$

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o the level set approach:

$$\phi^{n+1}(x_i) = \phi^n(x_i) - \Delta t^n V \partial_x \phi$$

Finite volume method?

 $\nabla \cdot \vec{V} = 0$

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 $\partial_t \phi + \vec{V} \cdot \nabla \phi = 0$

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Finite volume method!

 $\nabla \cdot \vec{V} = 0$

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 $\partial_t \phi + \nabla \cdot (\phi \vec{V}) = 0$

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 $\frac{\text{Finite volume method}}{\int\limits_{t^n} \int\limits_{\Omega_i} \left(\partial_t \phi + \nabla \cdot (\phi \vec{V}) \right) \, dx \, dt = 0$



<u>Finite volume method - analytical formulation</u> $\int_{\Omega_{i}} \phi(t^{n+1}) = \int_{\Omega_{i}} \phi(t^{n}) - \int_{t^{n}} \int_{\partial\Omega_{i}} \vec{n}_{i} \cdot \vec{V} \phi$

Finite volume method - discrete formulation

$$\phi_i^{n+1}|\Omega_i| = \phi_i^n |\Omega_i| - \Delta t^n \sum_j V_{ij} \phi_{ij}^{n+1/2}$$



 $\frac{\text{Finite volume method - analytical formulation}}{\int \limits_{\Omega_i} \phi(t^{n+1}) = \int \limits_{\Omega_i} \phi(t^n) - \int \limits_{t^n} \int \limits_{\partial\Omega_i} \vec{n}_i \cdot \vec{V} \phi$

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Definition of values in nodes (e.g., for initial time) •

$$\phi_i^0 := \phi^0(x_i)$$



Consistency of finite volume scheme on a structured grid:

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o averaged nodal values?!

$$\phi_i^0 := \frac{1}{|\Omega_i|} \int_{\Omega_i} \phi^0(x) \, dx$$

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Consistency of finite volume scheme on a unstructured grid:

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o averaged nodal values?

$$\phi_i^0 := \frac{1}{|\Omega_i|} \int_{\Omega_i} \phi^0(x) \, dx$$

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Finite volume scheme on a nonuniform grid might be inconsistent

JANSIA R. ANTES





Cockburn, Gremaud, Yang: A Priori Error Estimates for Numerical Methods for Scalar Conservation Laws, 1996 (... non-consistent schemes can converge ...)

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Barth, Ohlberger: Finite volume methods: foundation and analysis, 2004

Finite volume scheme on a nonuniform grid can be consistent

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o nodal values!

AND THE REAL

$$\phi_i^0 := \phi^0(x_i)$$

<u>Finite volume method - analytical formulation</u> $\int_{\Omega_{i}} \phi(t^{n+1}) = \int_{\Omega_{i}} \phi(t^{n}) - \int_{t^{n}} \int_{\partial\Omega_{i}} \vec{n}_{i} \cdot \vec{V} \phi$

Finite volume method - discrete formulation

$$\phi_i^{n+1}|\Omega_i| = \phi_i^n |\Omega_i| - \Delta t^n \sum_j V_{ij} \phi_{ij}^{n+1/2}$$

Definition of gradients in nodes

 $\nabla \phi_i^n = ?$ • many choices



Finite volume method

$$\int_{\Omega_i} \phi(t^{n+1}) = \int_{\Omega_i} \phi(t^n) - \int_{t^n} \int_{\partial\Omega_i} \vec{n}_i \cdot \vec{V} \phi$$

Flux-based form



Finite volume method

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$$\int_{\Omega_i} \phi(t^{n+1}) = \int_{\Omega_i} \phi(t^n) - \int_{t^n} \int_{\partial\Omega_i} \vec{n}_i \cdot \vec{V} \phi$$

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Flux-based form

$$\phi_{i}^{n+1}|\Omega_{i}| = \phi_{i}^{n}|\Omega_{i}| - \Delta t^{n} \sum_{j} V_{ij} \phi_{ij}^{n+1/2}$$

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Finite volume method

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$$\int_{\Omega_i} \phi(t^{n+1}) = \int_{\Omega_i} \phi(t^n) - \int_{t^n} \int_{\partial\Omega_i} \vec{n}_i \cdot \vec{V} \phi$$

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Flux-based form

$$\phi_i^{n+1}|\Omega_i| = \phi_i^n |\Omega_i| - \Delta t^n \sum_j V_{ij} \phi_{ij}^{n+1/2}$$

Approximation of fluxes - 2nd order consistent scheme?

$$\phi_{ij}^{n+1/2} = ?$$

 $V_{ij} > 0$

http://www.math.sk/frolkovi

Finite volume method

$$\int_{\Omega_i} \phi(t^{n+1}) = \int_{\Omega_i} \phi(t^n) - \int_{t^n} \int_{\partial\Omega_i} \vec{n}_i \cdot \vec{V} \phi$$

Flux-based form

 $\phi_i^{n+1}|\Omega_i| = \phi_i^n |\Omega_i| - \Delta t^n \sum_j V_{ij} \phi_{ij}^{n+1/2}$

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Approximation of fluxes - 2nd order consistent scheme

 $\phi_{ij}^{n+1/2} = \phi_{ij}^n - \frac{\Delta t}{2} \vec{V}_i \cdot \nabla \phi_i^n$

 $V_{ij} > 0$ http://www.math.sk/frolkov

Finite volume method

$$\int_{\Omega_i} \phi(t^{n+1}) = \int_{\Omega_i} \phi(t^n) - \int_{t^n} \int_{\partial\Omega_i} \vec{n}_i \cdot \vec{V} \phi$$

Flux-based form

 $\phi_i^{n+1}|\Omega_i| = \phi_i^n |\Omega_i| - \Delta t^n \sum_j V_{ij} \phi_{ij}^{n+1/2}$

Approximation of fluxes - 2nd order consistent scheme

$$\phi_{ij}^{n+1/2} = \phi_i^n + (x_{ij} - x_i) \cdot \nabla \phi_i^n - \frac{\Delta t}{2} \vec{V}_i \cdot \nabla \phi_i^n$$

 $V_{ij} > 0$ http://www.math.sk/frolkovi

Finite volume method

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$$\int_{\Omega_i} \phi(t^{n+1}) = \int_{\Omega_i} \phi(t^n) - \int_{t^n} \int_{\partial\Omega_i} \vec{n}_i \cdot \vec{V} \phi$$

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Flux-based form

$$\phi_i^{n+1}|\Omega_i| = \phi_i^n |\Omega_i| - \Delta t^n \sum_j V_{ij} \phi_{ij}^{n+1/2}$$

Approximation of fluxes - Taylor expansion

$$\phi_{ij}^{n+1/2} = \phi_i^n + (x_{ij} - x_i) \cdot \nabla \phi_i^n + \frac{\Delta t}{2} \partial_t \phi_i^n$$

 $V_{ij} > 0$ http://www.math.sk/frolkovi

Conservation laws with source term

Prover as the section of the section

 $\partial_t \phi + \nabla \cdot \left(\phi \vec{V} \right) = \phi \nabla \cdot \vec{V}$

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Conservation laws with source term

$$\partial_t \phi + \nabla \cdot \left(\phi \vec{V} \right) = \phi \nabla \cdot \vec{V}$$

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Flux-based level set method

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$$\phi_i^{n+1} |\Omega_i| = \phi_i^n |\Omega_i| - \Delta t^n \sum_j V_{ij} \left(\phi_{ij}^{n+1/2} - \phi_i^{n+1/2} \right)$$

Numerical approximation of $\nabla \cdot \vec{V}$

$$\nabla \cdot \vec{V} \approx \sum_{j} V_{ij}$$



Conservation laws with source term

$$\partial_t \phi + \nabla \cdot \left(\phi \vec{V} \right) = \phi \nabla \cdot \vec{V}$$

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Flux-based level set method

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$$\phi_i^{n+1} |\Omega_i| = \phi_i^n |\Omega_i| - \Delta t^n \sum_j V_{ij} \left(\phi_{ij}^{n+1/2} - \phi_i^{n+1/2} \right)$$

Approximation of source

$$\phi_i^{n+1/2} = ?$$



Conservation laws with source term

$$\partial_t \phi + \nabla \cdot \left(\phi \vec{V} \right) = \phi \nabla \cdot \vec{V}$$

Flux-based level set method

$$\phi_{i}^{n+1}|\Omega_{i}| = \phi_{i}^{n}|\Omega_{i}| - \Delta t^{n}\sum_{j}V_{ij}\left(\phi_{ij}^{n+1/2} - \phi_{i}^{n+1/2}\right)$$

Approximation of source - first order scheme

$$\phi_i^{n+1/2} = \phi_i^n$$

Frolkovic, Mikula: Flux-based level set method: a finite volume method ..., ANM 2007

Conservation laws with source term

$$\partial_t \phi + \nabla \cdot \left(\phi \vec{V} \right) = \phi \nabla \cdot \vec{V}$$

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Flux-based level set method

$$\phi_i^{n+1} |\Omega_i| = \phi_i^n |\Omega_i| - \Delta t^n \sum_j V_{ij} \left(\phi_{ij}^{n+1/2} - \phi_i^{n+1/2} \right)$$

Approximation of source - second order scheme

$$\phi_i^{n+1/2} = \sum \alpha_{ij} \phi_{ij}^{n+1/2}$$

Frolkovic, Mikula: "High-resolution flux-based level set method", SIAM SJSC 2007

Conservation laws with source term

$$\partial_t \phi + \nabla \cdot \left(\phi \vec{V} \right) = \phi \nabla \cdot \vec{V}$$

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Flux-based level set method

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$$\phi_{i}^{n+1}|\Omega_{i}| = \phi_{i}^{n}|\Omega_{i}| - \Delta t^{n} \sum_{j} V_{ij} \left(\phi_{ij}^{n+1/2} - \phi_{i}^{n+1/2}\right)$$

Approximation of source - Taylor expansion

$$\phi_i^{n+1/2} = \phi_i^n + \frac{\Delta t}{2} \partial_t \phi_i^n$$

Conservation laws with source term

$$\partial_t \phi + \nabla \cdot \left(\phi \vec{V} \right) = \phi \nabla \cdot \vec{V}$$

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Flux-based level set method

$$\phi_{i}^{n+1}|\Omega_{i}| = \phi_{i}^{n}|\Omega_{i}| - \Delta t^{n} \sum_{j} V_{ij} \left(\phi_{ij}^{n+1/2} - \phi_{i}^{n+1/2}\right)$$

Approximation of source

$$\phi_i^{n+1/2} = \phi_i^n - \frac{\Delta t}{2} \vec{V}_i \cdot \nabla \phi_i^n$$

Frolkovic, Wehner: Flux-based level set method on rectangular grids ..., CVS 2008

Rotation of a circle

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2nd order ENO

Rotation of a circle





2nd order ENO

Flux-based LSM

<u>Single-vortex example</u> $\vec{V} = (-\partial_y \Psi, \partial_x \Psi), \quad \Psi = \frac{1}{\pi} \cos(\pi t/8) \sin^2(\pi x) \sin^2(\pi y)$



$\frac{\text{Single-vortex example}}{\vec{V} = (-\partial_y \Psi, \partial_x \Psi), \quad \Psi = \frac{1}{\pi} \cos(\pi t/8) \sin^2(\pi x) \sin^2(\pi y)$



Single-vortex example

o approximation of the area: left 100² nodes, right 200² nodes

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Single-vortex example

o the reconstructed gradient similar to Beam-Warming scheme

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Ι	N	Solution	EOC	Area	EOC
50	500	1.30E-2		-4.02E-2	
100	1000	4.43E-3	1.56	-3.29E-3	3.61
200	2000	1.43E-3	1.63	2.73E-3	0.27
300	3000	6.25E-4	2.05	1.68E-3	1.20
400	4000	3.35E-4	2.17	8.07E-4	2.55
500	5000	2.06E-4	2.18	3.44E-4	3.83
600	6000	1.39E-4	2.17	1.37E-4	5.05
700	7000	9.93E-5	2.17	5.86E-5	5.50
800	8000	7.44E-5	2.17	3.37E-5	4.14

Single-vortex example o 2nd order o 200x200 grid



Single-vortex example o 1st order o 200x200 grid



Local grid refinement and coarsening

States of the second





stand p. Alta River Step on

Conclusions

0

o flux-based level set method

o "simple"

- o MATLAB version available (Ch. Wehner)
- o Unstructured Grids (UG) shall be available
- o see math.sk/frolkovi for Quicktime version of this lecture