Computing Two-Phase Flows in Porous Media with a Two-Fluid Hyperbolic Model

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Many design and safety studies for pressurised water reactors, steam generators and condensers require to use the porosity concept in control volumes.We examine here the suitability of some Finite-Volume schemes in order to compute a two-fluid hyperbolic model in a porous medium. Emphasis is put on the behaviour of Finite Volume schemes when the computational domain contains a sharp porosity variation. This is of course mandatory when aiming at coupling free and porous codes (see [1], and [7] that focuses on a specific homogeneous model). Properties of schemes and some more details can be found in [5].

A two-fluid hyperbolic model in porous media

The two-fluid hyperbolic model to simulate two-phase flows in porous medium is introduced in [8]. It is the counterpart of the models examined in [2, 3, 9]. Smooth solutions obey a physically relevant entropy inequality, and the governing set of equations admits unique jump conditions through single waves. Standard notations are used here: W_ε is the conservative variable, α_k is the void fraction of phase k, and ε stands for the porosity.

$$W_\epsilon^t = (\epsilon m_k, \epsilon m_k U_k, \epsilon \alpha_k E_k) \text{ in } \mathbf{R}^6$$

$$\begin{cases} \partial_t(\epsilon) = 0 \\ \partial_t(\alpha_2) + V_I \partial_x(\alpha_2) = \phi_2(\epsilon, \alpha_2, W_\epsilon) \\ \partial_t(\epsilon m_k) + \partial_x(\epsilon m_k U_k) = 0 \\ \partial_t(\epsilon m_k U_k) + \partial_x(\epsilon m_k U_k^2) + \epsilon\alpha_k \partial_x(P_k) + \epsilon(P_k - P_I)\partial_x(\alpha_k) = \epsilon I_k(\epsilon, \alpha_2, W_\epsilon) \\ \partial_t(\epsilon\alpha_k E_k) + \partial_x(\epsilon\alpha_k U_k(E_k + P_k)) + \epsilon P_I \partial_t(\alpha_k) = \epsilon V_I I_k(\epsilon, \alpha_2, W_\epsilon) \end{cases}$$

Closure laws for the source terms are the following, using: $(V_I, P_I) = (U_2, P_1)$

$$\phi_2 = \alpha_1\alpha_2(P_2 - P_1)/\tau_P \qquad \phi_1 + \phi_2 = 0$$

$$I_k = (-1)^k \frac{m_1 m_2}{(m_1 + m_2)}(U_1 - U_2)/\tau_U \qquad I_1 + I_2 = 0 \qquad E_k = \rho_k(e_k(P_k, \rho_k) + \frac{1}{2}U_k^2)$$

Three distinct Finite Volume schemes

The first Finite Volume scheme corresponds to the Rusanov scheme (R), the second scheme (MR) corresponding to a slight modification of the latter. The third scheme nicknamed WBR is a straightforward extension of the scheme [10]. It relies on the well-balanced strategy proposed in [6], though numerical fluxes are much simpler. If Z and f(Z) denote:

$$Z^t = (\alpha_2, m_k, m_k U_k, \alpha_k E_k)$$
$$f(Z)^t = (0, m_k U_k, m_k U_k^2 + \alpha_k P_k, \alpha_k U_k(E_k + P_k))$$

the approximate solution that is computed by the WBR scheme:

$$h_i(Z_i^{n+1} - Z_i^n) + \Delta t^n (F_{i+1/2,-}^{WBR}(Z_l^n, \epsilon_l) - F_{i-1/2,+}^{WBR}(Z_l^n, \epsilon_l)) + \Delta t^n (NCT)_i^n = 0$$

requires the following numerical fluxes :

$$F_{i+1/2,-}^{WBR}(Z_l^n, \epsilon_l) = (f(Z_i^n) + f(Z_{i+1/2,-}^n) - r_{i+1/2}^n (Z_{i+1/2,-}^n - Z_i^n))/2$$
$$F_{i-1/2,+}^{WBR}(Z_l^n, \epsilon_l) = (f(Z_i^n) + f(Z_{i-1/2,+}^n) - r_{i-1/2}^n (Z_i^n - Z_{i-1/2,+}^n))/2$$

and an evaluation of non-conservative terms NCT as follows:

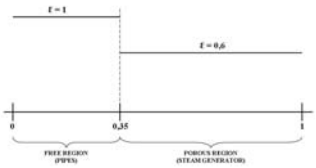
$$NCT_i^n = (\ (V_I)_i^n (\Delta\alpha_2)_i^n,\ 0,\ -(P_I)_i^n (\Delta\alpha_k)_i^n,\ -(P_I)_i^n (V_I)_i^n (\Delta\alpha_k)_i^n\ )$$

Setting: $$(\Delta\alpha_k)_i^n = (\overline{\alpha}_k)_{i+1/2}^n - (\overline{\alpha}_k)_{i-1/2}^n$$

The computation of interface states $Z^n_{i+1/2,-}$ and $Z^n_{i-1/2,+}$ requires solving two non-linear scalar equations at each cell interface (see [5]), thus enforcing the preservation of Riemann invariants of the steady wave through the interface. The WBR scheme enables to preserve equilibrium solutions –this is not true for R and MR schemes-, and its stability is ensured by a CFL-like condition. Following [7], some counterpart of the latter scheme which relies on an approximate well-balanced Godunov scheme may be exhibited.

Test case 1 : A loss of coolant accident

The first test-case corresponds to a rough representation of a loss of coolant accident, focusing on the propagation of the rarefaction wave that hits the free/porous interface separating the pipes and the steam generator.



Figures 1 and 2 show the behaviour of two Riemann invariants of the steady wave (enthalpy and entropy) for R scheme (blue line), MR scheme (red line), and WBR scheme (dark dashed line) respectively.

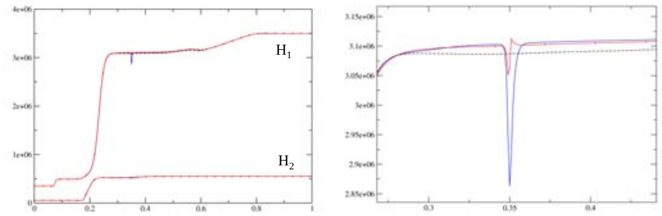


Fig 1: Enthalpies H_1 and H_2 (left). Zoom on the free/porous interface for H_1 (right)

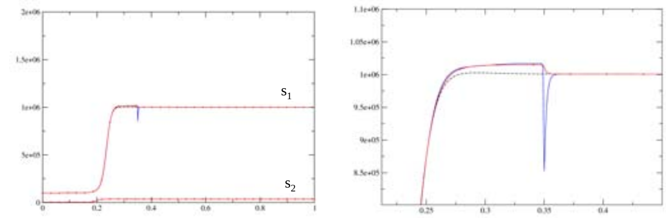
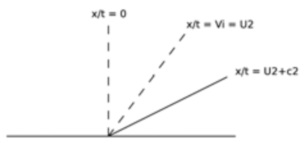


Fig 2: Entropies S_1 and S_2 (left). Zoom on the free/porous interface for S_1 (right).

Test case 2 : A classical one-dimensional Riemann problem

This second test-case corresponds to a classical one-dimensional Riemann problem, where left and right states of porosity are 1 and 0.6 respectively. Figure 3 provides the measure of the L^1 norm of the error, focusing on the WBR scheme.It enables to retrieve the expected rate of convergence ½ that is enforced by the two-contact discontinuities.



Sketch of the wave pattern of the 1D Riemann problem.

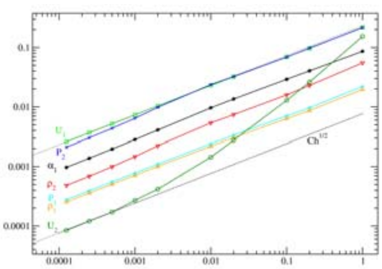


Fig 3: L^1 norm of the error for all state components (focusing on WBR scheme).

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